GENERATION OF SPATIAL LIGHT DISTRIBUTIONS

PhD dissertation

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Table of contents

1 Acknowledgements ................................................................................................................. 4

2 Theses ...................................................................................................................................... 5

2.1 Introduction ......................................................................................................................... 5

2.2 Stripe illuminator based on LED array and parabolic mirror for active triangulation sensors used on mobile robots ................................................................. 5

2.2.1 Description of the problem .............................................................................................. 5

2.2.2 The construction and properties of the stripe illuminator ................................................. 6

2.2.3 Summary of new results in technical sciences ............................................................... 7

2.3 Theoretical and numerical examination of the stripe illuminator made of a linear emitter placed at the focal line of a cylindrical lens to optimise the operation in external space .................................................................................. 8

2.3.1 Description of the problem .............................................................................................. 8

2.3.2 Examination of the stripe illuminator made of a linear emitter placed at the focal line of a cylindrical lens regarding operation in external space ............................................................................. 8

2.3.3 Summary of new results in technical sciences ............................................................... 9

2.4 Ternary phase-amplitude (+1,-1,0) modulation with transmission type twisted nematic liquid crystal displays for smoothing of spatial intensity distribution of Fourier holograms ............................................................................................................. 10

2.4.1 Description of the problem ............................................................................................ 10

2.4.2 Realisation of ternary phase-amplitude (+1,-1,0) modulation with transmission type twisted nematic liquid crystal displays .......................................................................................... 11

2.4.3 Summary of new results in technical sciences ............................................................... 12

2.5 Publications ....................................................................................................................... 13

2.5.1 Publications related to the theses ................................................................................... 13

2.5.2 Publications in optical engineering and science .......................................................... 14

3 General introduction .......................................................................................................... 15

4 Stripe illuminator based on LED array and parabolic mirror for active triangulation sensors used on mobile robots .................................................................................... 18

4.1 Introduction ....................................................................................................................... 18

4.1.1 General considerations ................................................................................................. 18
4.1.2 Stripe illuminators with lasers ................................................................. 19
4.1.3 Stripe illuminators with spectral lamps .................................................. 20
4.1.4 Stripe illuminators with LEDs ................................................................. 21

4.2 Operation principle ..................................................................................... 22

4.3 Design process by using an optical CAD software ...................................... 23
  4.3.1 The geometrical optical model of the LED ............................................. 24
  4.3.2 The calculation of the optimal parabolic shape and aperture ................ 26
  4.3.3 Tolerance analysis ................................................................................ 28

4.4 Realisation .................................................................................................... 30

4.5 Results and conclusions ............................................................................ 31
  4.5.1 Optical and geometrical characteristics .............................................. 31
  4.5.2 Eye Safety ............................................................................................. 34
  4.5.3 Results on a mobile robot .................................................................... 42

5 SNR analysis of the stripe illuminator based on linear emitter and cylindrical lens against solar irradiance ................................................................. 43
  5.1 Introduction ............................................................................................... 43
  5.2 The radiometric analysis of the stripe illuminator .................................... 45
    5.2.1 Definition of source parameters ....................................................... 45
    5.2.2 Radiance distribution at the exit aperture of the cylindrical lens ........ 46
    5.2.3 Calculation of the irradiance behind the lens .................................... 50
    5.2.4 Lens parameters yielding maximal total light power ......................... 53
  5.3 SNR against sunshine for sources with different types of emitters ........... 53
    5.3.1 Definition of SNR ............................................................................ 54
    5.3.2 SNR applying LED array ............................................................... 55
    5.3.3 SNR applying laser diode array ..................................................... 57
    5.3.4 SNR applying incandescent source .............................................. 60
  5.4 Conclusions .............................................................................................. 63

6 Ternary phase-amplitude modulation with transmission type twisted nematic liquid crystal displays for Fourier holographic data storage ......................... 65
  6.1 Introduction .............................................................................................. 65
  6.2 Derivation of the Jones matrix describing both the polarisation and phase modulation of a transmission type twisted nematic LC cell ......................... 67
6.3 Generation and detection of arbitrary elliptical polarisations .................73
6.4 TPA modulation setup of Jang and Shin......................................................76
6.5 Ternary Phase-Amplitude modulation applying transmission type twisted 
  nematic LC cell .................................................................................................77
  6.5.1 Ternary phase-amplitude modulation with $\Gamma=5.2$ .................................. 77
  6.5.2 Ternary phase-amplitude modulation with $\Gamma=6$ ..................................... 80
  6.5.3 Ternary phase-amplitude modulation with $\Gamma=6.6$ ................................. 82
6.6 Fourier plane homogenization with test images .........................................83
6.7 Summary and conclusions .........................................................................88

7 Appendix I .....................................................................................................89
  7.1 Virtual image of a single LED chip ...............................................................91
  7.2 The virtual image of the whole LED array ..................................................92

8 Appendix II ..................................................................................................94

9 References .....................................................................................................98
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2 Theses

2.1 Introduction
A spatial light distribution can be characterised by the spatial distributions of the following physical parameters depending on the coherence properties of the light: field vector, intensity, mutual intensity, polarisation, spectral power density, radiance. As my scientific theses concern mainly practical applications I do not consider the different possibilities of mathematical interpretation of spatial light distributions with more details.

Different optical applications require the generation of spatial light distributions of different physical properties. E.g. a very general task of technical optics is the generation of a light spot with very small geometrical size that is applied in many fields as optical data storage, laser surgery, laser material processing, etc. The application of movie and image projection requires spatial modulation of the spectral intensity of light according to the image to be projected. Since in most of the cases the light emitted by the primary sources does not meet the requirements of the applications, hence additional optical elements are needed for the modification of the light radiation. Such an equipment consisting of one or more primary light emitters and some optical elements for the modification of the properties of the emitted light can be called a light source or a light modulator. A spatial light distribution suiting to the requirement of a specific application can be obviously created on many different ways, which raises the problem of finding the technically and economically optimal solutions. Furthermore the steady development of technology enables to find more and more optimal solutions from time to time.

The first and second groups of theses are related to stripe illuminators for robotic applications, where the spatial modulation of the intensity and spectral intensity of light is required. The third groups of theses are related to spatial light modulation for holographic data storage applications, where the spatial modulation of the complex amplitude of polarised coherent light waves are considered.

2.2 Stripe illuminator based on LED array and parabolic mirror for active triangulation sensors used on mobile robots

Closely related publications: 1 and 2 of the list given in point 2.5.

2.2.1 Description of the problem
The light sources illuminating only a stripe of space are called stripe illuminators. An ideal light-stripe is such a spatial light distribution in which the light intensity is constant inside a certain three
dimensional stripe of space and zero outside of it. The most important application area of these light sources are the active triangulation sensors with structured illumination, which are also applied as obstacle recognition sensors of autonomous mobile robots (AMR). The target parameters of the developed stripe illuminator were specified according to the requirements of an obstacle recognition sensor developed for an experimental AMR of Siemens (ROAMER) and they are the following: 2m illumination range; stripe width of 6-8 cm; highest possible light intensity to detect objects with low reflexivity; high spectral intensity and simple capability of temporal modulation yielding high SNR against the background illumination in internal space; eye safety; simple construction for cheap production.

2.2.2 The construction and properties of the stripe illuminator

The above requirements can be fulfilled with a stripe illuminator constructed of a linear array of high power LEDs and a cylindrical parabolic mirror. The linear LED array is placed on the focal line cylindrical parabolic mirror and the LEDs are illuminating in the direction of the central line of the parabolic mirror. The light rays emerging from the LEDs are collimated in one direction by reflecting on the cylindrical parabolic mirror. An additional rectangular aperture placed at the plane of the LED array blocks the “off-axis” rays after reflecting on the mirror. This construction supplies very high total light power and intensity because of the large number of applied high power LEDs. The narrow bandwidth of the light emitted by the LEDs enables the spectral filtering of background illumination by applying an optical band-pass filter matched to the emission of the LEDs in the detecting optics. The application of cheap elements makes the stripe illuminator economically reasonable too.

I built a geometrical optical model of the selected high power LED (Siemens SFH487P, 880nm central wavelength) in an optical CAD system and designed a configuration (parabolic shape, distance between LEDs and mirror, size of rectangular aperture) optimally realising the above listed requirements. I made the tolerance analysis of the LED mounting that resulted tolerances of 0.2 mm for the positioning and 0.3° for the orientation.

The following simple and economic solution was found to realise the designed light source. The calculated parabolic shape was cut into thin metal plates by a precision laser material processing machine. More such metal plates were fixed side by side in an Al box and a flexible mirror foils was stretched onto the parabolic edges of the metal plates by springs. The accurate positioning and orientation of the LEDs were solved by a special assembling and holding frame.

The technical parameters of the realised stripe illuminator are the following. The stripe width is 75 mm (FWHM) at 2 m distance from the source. The intensity of the stripe are 5.5 mW/cm², 1.1 mW/cm², 0.6 mW/cm² at 0.1m, 1m and 2m distances respectively. The spectral intensity of the source was measured to be 69, 14 and 7.5 µW/cm²/nm at 0.1m, 1m and 2m distances respectively. Comparing these values with spectral intensities measured in internal spaces SNRs between 140 (room illuminated by artificial light) and 7 (room illuminated by strong sunshine) can be obtained.
depending on the intensity of the background illumination. The operation of the stripe illuminator was tested in a special active triangulation sensor. In internal space the light source supplied detectable signal even for objects of very small reflectivity, as black objects or objects made of transparent glass. According to the tests all types of objects except fully transparent or reflective surfaces of optical quality could be detected by the light source in internal space. By increasing the numerical aperture of the detecting optics and the detection time surfaces of optical quality may be also detected by the light source. The optical sensor was mounted onto an AMR to test the operation of the light source. The AMR could fulfil all of its navigation tasks by relying only on the signals of the optical sensor about the obstacles lying in its environment. I analysed the eye safety of the stripe illuminator based on physical principles and valid technical norms. Both analyses resulted that the stripe illuminator is eye safe.

2.2.3 Summary of new results in technical sciences

I. I recognised and justified with numerical calculations that the light source constructed of a cylindrical parabolic mirror, a linear LED array placed at the focal line of the mirror and a transmitting aperture of rectangular shape placed at the plane of the LED array realises a high power, eye safe stripe illuminator by which a stripe of less than 9 cm stripe width is illuminated within a distance range of 0-2 m. I prove with test measurements that the intensity and spectral intensity of the light stripe are high enough even for the detection of objects with low reflectivity in internal space thus the stripe illuminator can be effectively applied in obstacle recognition sensors based on active optical triangulation. To reach the stripe width of 9 cm the LEDs should be positioned with 0.2 mm tolerance and the orientated with 0.3° tolerance, which are technically feasible requirements. These tolerance values were calculated in an optical CAD system based on a geometrical optical model of the LED.

II. I recognised that a technically and economically effective solution of this stripe illuminator is a flexible mirror foil stretched onto metal plates having the required parabolic shape; the accurate positioning of the LEDs can be solved with a special mounting and holding frame. I prove with test measurements that the realised stripe illuminator supplied 75 mm stripe width (FWHM) within the 0-2 m distance range. As the light source of an optical triangulation sensor the stripe illuminator supplied signals high enough even for detecting objects with very low reflectivity (black objects, glass objects). The SNR against the background illumination varied between 140 and 7 depending on the intensity of background light. I justified with radiometric calculations and measurements prescribed by the valid standards that stripe illuminator is eye safe.
2.3 Theoretical and numerical examination of the stripe illuminator made of a linear emitter placed at the focal line of a cylindrical lens to optimise the operation in external space

Closely related publications: 3 of the list given in point 2.5.

2.3.1 Description of the problem

These theses consider the generation of such spatial light distributions that allow active triangulation sensors to operate in external space. The most important noise source for the external operation of these sensors is the intense background light caused by sunshine. The most important condition of efficient operation in external space is that spectral intensity of the stripe illumination should be high enough compared to the spectral intensity of terrestrial solar illumination, consequently the spatial modulation of spectral intensity is required.

2.3.2 Examination of the stripe illuminator made of a linear emitter placed at the focal line of a cylindrical lens regarding operation in external space

I examined in which way stripe illuminators supplying high enough SNRs against solar illumination can be realised by the construction of a cylindrical lens and a linear emitter (in reality rectangular, but one edge is much shorter than the other one) placed at the focal line of the lens. This construction is very effective for realising high power stripe illumination because of the large surface size of the rectangular emitter. Such a light source can be constructed of many different emitter types as LED array, laser diode array and incandescent source, each of them have different physical sizes and angular emission characteristics. My main aim was to set up a general theoretical model by which the illumination of the different practical stripe illuminators can be described, their external space operation can be optimised and the different practical configurations can be compared. Firstly I built a radiometric model of the optical system assuming a spatially invariant rectangular emitter of arbitrary angular radiation distribution and an ideal cylindrical lens. The model resulted an integral by which the (spectral) intensity can be calculated in any point of the illuminated stripe. I determined the optimal focal length and numerical aperture of the cylindrical lens that couples the maximal light power into a stripe of given width from a rectangular emitter of a given geometrical size. By the numerical evaluation of the radiometric integral I examined the SNRs against solar illumination.
applying different practical rectangular emitters as linear LED chip array, linear array of packaged LEDS, laser diode array and tungsten incandescent sources. Stripe illuminators with four different geometrical parameters (see Table) were examined by the emitter types listed above.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Illuminating distance</th>
<th>Stripe width</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 m</td>
<td>0.01 m</td>
</tr>
<tr>
<td>2</td>
<td>1 m</td>
<td>0.05 m</td>
</tr>
<tr>
<td>3</td>
<td>2 m</td>
<td>0.01 m</td>
</tr>
<tr>
<td>4</td>
<td>2 m</td>
<td>0.05 m</td>
</tr>
</tbody>
</table>

The SNR was defined as ratio of the spectral intensity of the stripe at maximal distance and the spectral intensity of direct solar radiation. The terrestrial solar spectrum published by the American Society for Testing and Materials were applied in the calculations.

By applying a linear emitter made of linear LED chip array SNRs over one were obtained with chips emitting at 450nm, 660nm and 950nm central wavelengths for all the configurations of the table. The 880nm LED chip array supplied SNR over one for the configurations 1,2 and 4. By applying a linear array of 950nm packaged LEDS SNRs over one were obtained for all the configurations except the 3rd one. SNR over one was obtained also by applying 660nm packaged LEDs for configuration 2. By applying a linear array of 5mW packaged laser diodes SNRs over 0.7 were obtained for all configurations at all the investigated wavelengths (405, 635, 785, 840, 950 and 1550nm). The 405 and 950nm laser diodes supplied SNRs over one for all the configurations. By increasing the power of the laser diodes the SNRs can be further increased. The SNRs were calculated also for tungsten linear sources of 2500K and 3000K temperatures. SNR over one were obtained for the 2nd configuration of the table at wavelengths over 800nm with the 2500K tungsten source and over 540nm with the 3000K one. These sources can be effectively applied if a stripe illuminator with very high total power and broad spectral band is required. For the configurations 1,3,4 the SNR was over one only in very narrow spectral bands (in the solar minima), thus the application of these sources is not advantageous because of the low power efficiency.

2.3.3 Summary of new results in technical sciences

I. I examined and modelled the stripe illuminator made of an ideal cylindrical lens and a spatially invariant rectangular emitter of arbitrary angular radiation distribution placed at the focal line of the lens to examine its effectiveness in external space measurements. I established a radiometric integral by which the (spectral) intensity of the illuminated stripe can be calculated for rectangular emitters of arbitrary angular radiation distribution and an ideal cylindrical lens of arbitrary focal length and aperture size. This model is able to
describe practical stripe illuminators realised with different rectangular sources as LED array, laser diode array and incandescent source.

II. I defined the SNR as the ratio of the spectral intensities of the stripe illuminator and of direct sunshine. By the numerical evaluation of the radiometric integral I calculated the obtainable SNRs for stripe illuminators applying LED chip arrays, arrays of packaged LEDs and laser diode arrays operating at various wavelengths; I also calculated the SNRs using 2500K and 3000K tungsten rectangular sources. According to the obtained SNRs I selected those emitter types by which SNRs over one could be obtained: LED chip arrays of 470, 525, 660, 880, 940 and 950nm wavelengths; packaged LED arrays of 660 and 950 nm wavelengths (very simple and economical solutions!); array of packaged 5mW laser diodes of 405, 635, 785, 840, 950, 1050nm wavelengths; 2500K and 3000K tungsten incandescent sources in different spectral bands defined by the geometry of the illuminated stripe. By these calculations I showed that the stripe illuminator based on a cylindrical lens and a linear emitter can be effectively applied in external space measurements.

2.4 Ternary phase-amplitude (+1, -1, 0) modulation with transmission type twisted nematic liquid crystal displays for smoothing of spatial intensity distribution of Fourier holograms

Closely related publications: 4 and 5 of the list given in point 2.5.
Other related publications: 6-12 of the list given in point 2.5.

2.4.1 Description of the problem

These theses concern with the spatial modulation of the complex amplitude of linearly polarised coherent light for holographic data storage. In Fourier holographic data storage data images of several bytes of information are stored in holograms. The data images are usually created by spatial light modulators (SLM), the Fourier transformation is made by an objective. A basic problem of Fourier holography is that intensity of the zero spatial frequency component can be several magnitudes higher than the average intensity of other frequencies. Storing such a hologram does not optimally utilise the dynamic range of the storing material. A well known method to destruct the zero frequency component is to give random phase modulations to the different SLM pixels. The conventional solution for this method is that the SLM is imaged onto a random phase mask with same pixel number and geometry as the SLM by an objective of sub-pixel distortion. Both the design and fabrication of an objective with sub-pixel distortion and the 6 axes alignment required for the pixel matched positioning of the SLM and phase mask are complicated technical tasks, thus this is a very
uneconomical solution. Furthermore for those data images in which the bright SLM pixels well correlate with the phase mask pixels of similar phase shifts the zero frequency component is not destructed, thus they should be prevented by coding techniques that leads to data capacity losses. Because of the above described reasons it would be very advantageous to realise the spatial amplitude and phase modulation of light required for the generation of Fourier holograms with smoothed intensity distributions with a single SLM pixel.

2.4.2 Realisation of ternary phase-amplitude (+1,-1,0) modulation with transmission type twisted nematic liquid crystal displays

My research work concerned with the problem of realising the required complex amplitude modulation of light by applying a single pixel of a transmissive twisted nematic LCD. The simplest answer for the described problem is the ternary phase-amplitude (TPA) modulation containing three states with complex amplitude modulations of +1,-1 and 0. This modulation scheme is already known in the field of correlation filters.

The complex amplitude modulation of an ideal transmission type twisted nematic LC cell can be described by a Jones matrix assuming that the LC molecules are uniformly twisting within the LC medium. According to the model the LC cell has two important parameters: the phase retardation that can be modulated between a maximal value and zero by applying electrical field to the cell, the maximal value is taken at zero field; and the twist angle of the LC molecules within the cell that is invariant to the applied electrical field and approximately equal to ±90° for commercially available LCDs. The elements of Jones matrix are non-linear functions of the phase retardation and the twist angle. In order to obtain TPA modulation I placed the LC cell between an “elliptical polariser” and an “elliptical analyser”\(^\odot\). Based on the Jones matrix I built a numerical model by which the complex amplitude modulation of the LC cell could be evaluated for arbitrary elliptically polarised illumination and light detection. An elliptical polarisation can be described by two independent parameters (e.g. ratio of major and minor axes and direction of major axis) thus the illuminated and detected elliptical polarisations can be described by four independent parameters. The complex amplitude modulation of the LC cell was calculated as a function of the phase retardation in points of the four dimensional parameter-space that describes the incident and detected elliptical polarisations. The elliptical polariser is an optical element used to generate arbitrary elliptical polarisation. One realisation is a sequence of a revolving linear polariser and a revolving quarter wave plate. An elliptical analyser is capable of transmitting one elliptical polarisation and blocking the orthogonal elliptical polarisation. It can be realised by the sequence of a revolving quarter wave plate and a revolving linear polariser.

\(^\odot\)An elliptical polariser is an optical element used to generate arbitrary elliptical polarisation. One realisation is a sequence of a revolving linear polariser and a revolving quarter wave plate. An elliptical analyser is capable of transmitting one elliptical polarisation and blocking the orthogonal elliptical polarisation. It can be realised by the sequence of a revolving quarter wave plate and a revolving linear polariser.
polarisations for which the LC cell realises TPA modulation were found by computer search. The searching condition was that the calculated complex amplitude modulation function must contain three points of complex amplitudes $A_1,A_2,\varepsilon$ with the following conditions: $0.95<|A_1/A_2|<1.05; 0.95\pi<\arg(A_1/A_2)<1.05\pi; |A_1|^2/|\varepsilon|^2>15$. As the maximal phase retardation of an LC cell depends on the wavelength and different LCDs have different maximal phase retardations as well, hence the computer search was executed for more different maximal phase retardation values. By increasing the maximal phase retardation better modulation capabilities are expected. Applying an LC cell with 5.2 phase retardation TPA modulation can be obtained with an amplitude transmission of $|A_1|=|A_2|=0.5$, namely 25% of the incident light power can be utilised. An LC cell with a phase retardation of 6 supplies TPA modulation with $|A_1|=|A_2|=0.6$, thus 36% of the light power is utilised. An LC cell with a phase retardation of 6.6 yields TPA modulation with $|A_1|=|A_2|=0.9$, thus 81% of the light power is utilised. In the last case $|\varepsilon|=0.16$ resulting a contrast ratio of $(0.9/0.16)^2=32$ between the intensities of the bright and dark states. The intensity contrast ratios in the first two cases were 15 and 16.

I made test measurement to verify the calculated results experimentally applying a transmission type twisted nematic LCD of Sony (LCX017DLT). The TPA modulation characteristics calculated for phase retardations of 5.2 and 6 were measured at 532 and 473 nm wavelengths respectively. The measured complex modulation curves were consistent with the calculations, TPA modulation could be experimentally realised with similar parameters as the calculations predicted. The intensity of the Fourier plane were compared for TPA and normal intensity modulation of 2 states applying test data bit patterns. Applying conventional intensity modulation of 2 states the ratio of peak and average intensities of the Fourier plane was 200:1, while in the case of TPA modulation it was 8:1.

### 2.4.3 Summary of new results in technical sciences

I. I recognised that TPA modulation can be realised with transmission type twisted nematic LC cells of approximately ±90° twist angle, if the LC cell is placed between an elliptical polariser and an elliptical analyser and the configurations of the elliptical polariser and analyser are optimised to the phase retardation of the LC cell.

II. I recognised that by increasing the phase retardation of the LC cell and applying an elliptical polariser and analyser optimised to the increased phase retardation the properties of TPA modulation (transmission of bright states, contrast ratio between intensities of bright and dark states) can be enhanced. I showed with numerical calculations and test measurements that with phase retardations of 5.2, 6 and 6.6 the obtainable amplitude transmissions of TPA modulation are 0.5, 0.6 and 0.9 respectively. The contrast ratios between the intensities of bright and dark states are 15,16 and 32 respectively for the above phase retardation values.
2.5 Publications

2.5.1 Publications related to the theses


2.5.2 Publications in optical engineering and science


15. Domján L, Szarvas G, Mike Sz: Multiple imaging arrangements for head-mounted displays, US patent application, 18 November 2003,
3 General introduction

Optical devices are very widely used in consumer products and in industrial and scientific applications as well. The most important consumer product areas based on optical technologies are lighting products (e.g. room or car lamps), data storage devices (CD,DVD,MO), communication systems, photographic and video systems and eye correction glasses. I can list only some of the industrial applications since there are very many of them: microscopes, machine controlled inspection systems, laser material processing, lithographic systems, position measurements applied in e.g. geodesy and robotic applications. Optical systems are also utilised in various fields of nature science as biology, medicine, astronomy, chemistry, material and environmental sciences, etc.

My PhD thesis relate to the establishment of spatial light distributions optimised to meet the requirements of special applications. A device that creates a certain spatial light distribution can be called a light source or a light modulator. Such a device contains usually at least one light emitter and some optical elements (lenses, mirrors, filters, polarizers, etc ) that modify the illumination to meet the requirements of the special optical applications. Some examples of applications requiring special spatial light distributions are: spatial modulation of light intensity is required in film and image projection applications; the focusing of light into a very small spot is a general problem in many applications as data storage, laser surgery, laser material processing; the establishment of spatial light structures as stripes, gratings, circular patterns is required in position sensing using structured illumination. Such spatial light distributions can be usually created on many ways because of the wide variety of light emitters (e.g. lasers, LEDs, gas lamps, incandescent sources) and light modulating tools. Hence finding an optimal solution regarding technological and economical objectives is a non-trivial issue. Furthermore the technological progress in the fields of light emitters, light modulating elements, modelling tools and computer technology enables to find more optimal solutions from time to time for a given application.

Chapters 4 and 5 deal with the establishment of stripe illuminators for robotic applications. Robot manipulators are already applied in many fields of industrial systems as e.g. mounting, painting, drilling, food production. Another very interesting field of robotics are the autonomous mobile robots (AMR) which are very intensively researched both in university and industrial laboratories. A typical AMR is a vehicle capable of moving autonomously within an environment that is either partially known or absolutely unknown by it. Such AMRs can be applied both in industrial and consumer applications as cleaning robots, transport robots, agriculture robots, etc. In order to make accurate and safe movements without hitting the humans and the objects surrounding the robot sensors are required to recognise the objects and to measure their positions. There are many different types of AMRs developed by universities and industrial research laboratories such as the ROAMER of Siemens and the human like robots of Honda, which are not yet widely applied in industrial or consumer applications. From pure technical point of view today’s AMRs are capable of solving different tasks
without threatening the health or material property of human beings, but the high costs of the reliable and accurate sensors prevent the widespread application of this technology both in the consumer and the industrial areas. Hence huge efforts are made to develop low cost sensors for reliable obstacle recognition and position measurement. Chapter 4 of my dissertation discusses a low cost stripe illuminator designed for active vision systems applied in AMRs. The stripe illuminator is based on a cylindrical parabolic mirror and a linear LED array placed at the focal line of the mirror. The stripe illuminator effectively worked on the AMR of Siemens (ROAMER) as part of a low cost stripe illumination based active vision sensor system. The design of this stripe illuminator concentrated mainly on two parameters: on the irradiance and spectral irradiance of the light stripe. High irradiance was required to detect objects with low reflectivity and high spectral irradiance was needed to achieve good signal to noise ratio (SNR) against the background illumination. The achieved spectral irradiance was high enough to obtain excellent SNRs against the background illumination of indoor environments, but in outdoor environment with direct sunshine the source could not be applied. Chapter 5 consider stripe illuminators based on a cylindrical lens and a rectangular emitter placed at the focal line for outdoor applications. The examination focuses on the spectral irradiances of the stripe illuminators realised with different rectangular emitters such as LED array, laser diode array and incandescent sources of different temperatures. By comparing the obtained spectral irradiances with the spectral irradiance of direct sunshine the achievable SNRs against sunshine are calculated. The spectral irradiances and the SNRs are calculated by applying the means of radiometry and the theoretical radiation characteristics of the analysed emitter types. Chapter 6 deals with establishing special spatial light distribution for holographic data storage in which the complex amplitude of light is spatially modulated. In holographic data storage the storage unit is a hologram that can store a data image containing up to megabits of data, in contrast to conventional digital optical data storage (CD,DVD) in which the storage unit is a single spot storing usually only a single bit of information. The storage holograms can be written side by side in a single thin layer or can be written on a multiplexed way into volume of holographic storing material as well. The main advantage of holographic storage over the conventional digital optical storage techniques is that extremely high storage capacity can be obtained through volume multiplexing of holograms. According to theoretical calculations and experimental data e.g. in a multiplexed holographic disk having the same geometrical measures as a compact disk more than one terabytes of information can be stored. It is well known that similar or even higher amounts of data can be stored by semiconductor memories and magnetic hard drives as well. The main advantage of holographic and in general of optical disks over these concurrent technologies is that the cost of the storing medium is significantly lower. An other advantageous application area of this technology is high security data storage. By applying a spatially phase modulated reference beam during hologram recording a physical encryption of the stored data can be achieved. The data image stored in the hologram can be read only if the hologram is illuminated with the same phase modulated reference beam used at recording, thus the data specifying the phase modulation of the reference beam make an encryption key. Holographic data
storage has been intensively researched for 20-30 years but today there is no commercial product based on this technology. The lack of products is mainly caused by the expensive elements required in a holographic storage device such as coherent laser, spatial light modulator, CCD sensor and objectives corrected for wide object field. Hence technological improvements reducing the costs are essentially needed by holographic data storage to leave the laboratory. The data image stored in a hologram is displayed by a spatial light modulator (SLM) which is illuminated with a coherent light beam. In thin storing material the Fourier transform of the data image is usually stored to achieve optimal data density. In order to get rid of the high peak at zero frequency of the hologram the data image should be both amplitude and phase modulated. Conventional SLMs are optimised for modulating only the intensity of light to display images for the eye or a camera. The phase modulation is usually obtained by an external phase modulating mask having the same pixel number and geometry as the SLM and the two devices are imaged onto each other with sub-pixel resolution. Both an objective supplying sub-pixel distortion and the six axes alignment of the SLM and phase mask are expensive technologies. Chapter 6 considers the realisation of the required amplitude any phase modulation of light by applying a single twisted nematic liquid crystal SLM of transmission type, which is the most conventional SLM available on the market. By illuminating the SLM with elliptically polarized light and detecting elliptical polarization the required amplitude and phase modulation of light can be achieved. Since each SLM pixels can be operated independently the complex amplitude of light can be spatially modulated.
4 Stripe illuminator based on LED array and parabolic mirror for active triangulation sensors used on mobile robots

4.1 Introduction

4.1.1 General considerations

Stripe lighting sensors are widely used in robotics to measure the position of obstacles in the workspace of an autonomous mobile robot [1.][2.][3.]. They operate on the principle of illuminating a light stripe (usually horizontal) around the robot (see 4.1. Fig.), and this room area is surveyed by a camera. If an obstacle is hit by the light stripe, it scatters a portion of the illumination back, which causes a bright-spot in the image of the detecting camera. Through determining the position of the image spot by image processing, the position of the obstacle can be calculated, if we know the relationship between the image points and the points of the light stripe. The camera lens transforms the points of the room onto the 2 dimensional imaging device. Knowing the parameters of this transformation is necessary to make position measurements with a stripe lighting sensor.

In this paper we want to focus on the problem of finding an efficient and economic stripe illuminator for the above purpose.

The three main characteristics of a stripe illuminator are:

1. The irradiance of the illuminated light (measured in mW/cm²).
2. The spectral irradiance of the illuminated light (measured in mW/cm²/nm).
3. The geometrical width of the stripe.
We discuss first the influence of the irradiance. The detectability of objects with low reflectivity becomes better by increasing the irradiance. If the source is to be used on a mobile robot, which operates among people, it is a further requirement that the source must not injure the human eye. This limits the irradiance of the illuminated light depending on the wavelength and on the type of the source.

High spectral irradiance helps suppressing the disturbing effects of the background light. The background light in the environment of the robot is the main noise-source for a striping sensor [1.]. There are two possibilities for suppressing the effects of the disturbing background illumination. If the stripe illuminator emits just in a narrow spectral band (such as lasers and LEDs), then by using an optical bandpass filter which is fitted to the spectral band of the light source in front of the camera the background light can be supressed. Another filtering method can be used when the light source is pulsed, and two images, one with and one without stripe illumination are taken. By substracting the second image from the first, the signals originating from the background illumination cancel. If it is neccessary, both methods can be used simultaneously.

The width of the light stripe influences the achievable distance resolution of the sensor. The less the stripe width is, the better the achievable resolution becomes. Namely the localization of an object is based on the fact that it has to be on the intersection of the illuminated stripe and of the line, drawn from the corresponding image point through the center of the camera objective. By decreasing the width of the light-stripe, this intersection becomes shorter, therefore the resolution becomes better. However decreasing the stripe width has also a drawback, namely that the space covered by a light-stripe becomes narrower, and the number of stripe illuminators needed to cover the whole room increases.

4.1.2 **Stripe illuminators with lasers**

![Diagram of stripe illuminator with laser and cylinder-lens](https://via.placeholder.com/150)

Stripes Illuminator with laser and cylinder-lens

4.2. Fig.
The most widely used version for a stripe illuminator is a collimated laser and a cylindrical lens, where the cylindrical lens creates the needed light stripe (see 4.2. Fig.). The main advantages of this version are that the stripe width can vary between small (1mm) and large (10cm) values, and that the very small spectral bandwidth of lasers enables a very high spectral irradiance. In addition the electronic pulse modulation of semiconductor lasers is easy and finally, that the optics used is quite simple. The main drawbacks are: the high costs of lasers, and the very strict eye safety limits for laser sources [4.][13.], which may limit the application in a strongly illuminated environment.

4.1.3 **Stripe illuminators with spectral lamps**

Stripe illuminators can also be produced with different types of spectral lamps (e.g. gas lamps, incandescent lamps). Two common arrangements for creating light stripes with spectral lamps are shown in 4.3. Fig.. In the first version the stripe is produced by a cylindrical lens [5.]. In the second version the rays are first collimated by a lens and the parallel beam is projected onto a conic mirror having an axis parallel with the beam. The conic mirror reflects the rays in such a way that a light stripe normal to the cone-axis is produced. In order to collect a portion of those rays, which are not emitted in the direction of the lens, sometimes a back reflector used.

The main advantages of the application of spectral lamps in stripe illuminators are, that a very high irradiance, and a good spectral irradiance can be achieved. But it has also disadvantages. Since only a small portion of the total emitted light can be collected and collimated, therefore in order to achieve a high spectral irradiance (e.g. 10 μW/cm²/nm at 1m distance) a total light power of hundreds of Watts are needed for the whole stripe. The application of such high power lamps on a mobile robot is
inpractical. The simplest solution for periodical pulsing of such lamps at the frequency needed (>10 Hz) is mechanical chopping, which is also disadvantageous compared to electronic modulation.

4.1.4 **Stripe illuminators with LEDs**

Stripe illuminators can also be constructed using LEDs. The rapid progress of LEDs in the last decade enabled them to substitute for incandescent sources in applications, in which high irradiance is needed, as e.g. break-lamps of cars [6.][7.] or lasers in medical therapy[8.]. The main advantage of LEDs over spectral lamps is that the LEDs emit with higher spectral radiances (measured in mW/cm²/sr/nm). The technical parameters of an emitter (radiance, spectral radiance, ability of collimation), which are important when making a stripe illuminator of it, are superior for lasers compared to LEDs. According to [4.] the same eye safety regulations are as valid for LEDs as for lasers, but since the emitting area of LEDs (~0.4x0.4 mm) is much greater than the emitting area of laser diodes (~10x10 µm) LEDs are expected to supply better eye safety. A further advantage of LEDs compared to lasers is their lower costs. This can be very important in mass production, primarily when an array of sources is needed to be used in an application requiring a high total light power (>500mW). Since the power of a single LED (20 mW) or a cheap Laser Diode (10 mW) is not strong enough for a stripe sensor used in a well illuminated environment (>1W needed), the application of an array of cheap, high power LEDs is an effective and economical solution.

Creating a light-stripe with LEDs is a problem similar to creating a collimated beam, because in most cases using the "cylindrical form" of the collimating optics yields a stripe illumination. Collimation of LED light has been studied by some articles. Wilcken [9.] describes two optical setups for collimating the light of LEDs using conventional optical elements. In the first one an LED with a flat-truncated package is equipped with a pin-hole aperture that makes a semi point light source. The beams leaving the pin-hole are collimated with an objective lens of large Numerical Aperture (Effective Focus Length (EFL)=4.59mm, NA=0.5). In the second one, the beams of an LED with an integrated lens are focused into a pin-hole aperture by an objective lens of the same type, and the beams leaving the aperture are collimated with a second objective lens. Both versions give a collimated beam of high quality. The main disadvantage is the high loss of the emitted power (about 90%) because of the pin hole. Spigoulis describes in [10.] a complex optical element, which is proposed for collimating or focusing the light of semi-point sources as e.g. LEDs (see 4.4. Fig.). First the rays of the emitter enter slightly refracted into the element through a spherical surface. Then the rays close to the axis are collimated by refraction on an ellipsoidal surface, and the rays with larger angles are first totally internally reflected on a sloped paraboloidal surface and then refracted on a conical surface.
Parkin and Pelka present a Fresnel-like lens with a lot of facets for collimating or focusing the light of LEDs and lamps [11]. The rays at different heights are collimated through total internal reflection on different facets of the lens. The advantage of the last two collimators is that they collect the emitted rays in large acceptance angles, so a great portion of the total emitted power is collimated. By using the cylindrical form of any of these systems with a linear LED array on the focal line, a technically effective stripe illuminator could be created, but the production costs of a master, by which such cylindrical optical elements can be produced with complicated, non-circular cross-sections, are very high.

In summary I recognised that none of these solutions leads directly to an efficient, eye-safe and cost effective stripe illuminator.

**4.2 Operation principle**

My aim was to create a stripe illuminator that

- has a maximal full stripe width of 6-10cm (suitable for a number of robotic applications),
- supplies high irradiance,
- supplies high spectral irradiance,
- can be electronically modulated up to 10 Hz frequency.

A stripe illuminator meeting these requirements is shown in 4.5. Fig. It consists of a cylindrical mirror with parabolic cross-section, and of a linear array of high intensity IR LEDs. The focal line of the mirror is the line containing the focal points of all the parabolic cross sections, and the base line is the line containing all their base points. The operating principle is that a parabolic mirror collimates the beams of a point source, if it is located at the focal point. Therefore, if we put an array of point sources onto the focal line of a cylindrical parabolic mirror, then the beams, after reflection on the mirror, will be parallel with the plane, defined by the focal line and the base line. So we get a stripe illumination.

Since the LEDs can not be regarded as point sources, the effect of finite source size needs to be considered. The coordinate system applied in this chapter is shown in 4.5. Fig. The x axis is parallel
with the focal line of the mirror, $z$ is parallel with the axis of the parabolic cross sections, and $y$ is perpendicular to the illuminated stripe. The LEDs are positioned along the focal line of the mirror with an orientation, such that their optical axes are parallel with the $z$ axis.

The basic structure of the light source

4.3 Design process by using an optical CAD software

I decided to use IR LEDs because they have the maximum light power. High power IR LEDs are available with truncated flat surface or with integrated lens. I chose LEDs with flat surface, because the integrated lens has poor imaging quality, and this makes the LED less point-like. The Siemens SFH 487P LED was selected that has a continuous output light power of 22mW at 880nm central wavelength and a truncated T1 3mm plastic package. In order to evaluate the concept and to find the optimal parabolic geometry, a geometrical-optical model of the whole system in the CAD software OSLO Six was created.
4.3.1 The geometrical optical model of the LED

The structure of the Siemens SFH 487P LED without rays

The structure of the LED consists of an emitting chip, a metal back reflector of spherical shape, and a plastic package (see 4.6. Fig.). Through consultations with the manufacturer I got to know the exact structure and positioning of the chip and the internal back reflector. This information was necessary to simulate the light generation and extraction in the LED. The simulation model was the following: the light is generated in the pn-junction and propagates in all directions towards the different chip surfaces. The rays that arrive at the first surface with an incident angle less than the angle of total reflection refract at the surface and leave the chip. The rays with an incident angle over the angle of total reflection reflect and propagate towards the next surface of the chip. The rays that reach the substrate are stopped, because the substrate is not transparent to the generated light.

The model of the Siemens SFH 487P LED with emitted rays

4.7. Fig.
As 4.7. Fig. shows, a portion of the rays leave the chip through the front surface and propagates towards the front surface of the package. An other portion of the rays leaves the chip through the sides of the chip, mainly close to the junction, and propagates to the back reflector, and after reflecting or scattering propagates towards the front surface. 4.8. Fig. shows two camera images of the LED at 100mA forward current. In 4.8. Fig./a a microscope image is shown in which the axes of the LED and of the microscope were parallel. Because of the limited numerical aperture of the microscope 4.8. Fig./a shows the radiant intensity distribution of the chip and reflector for those rays that propagate close to the optical axis of the LED. As it can be seen in this case the most significant portion of the total emission comes from the front surface of the chip and a much less significant portion comes from the total surface of the back reflector. As the LED illuminates in a wide angular field another microscope image was taken with an arrangement in which the LED was tilted and the optical axes of the microscope and of the LED subtended about 30 degrees (see 4.8. Fig./b). In this image the brightness of the reflector is much closer to the brightness of the chip surface than in the previous image, thus in this tilted direction the reflector illuminates similar radiant intensity as the chip does. The appearance of these images can be explained quite well by the model. As 4.7. Fig. shows a portion of the rays leave the chip through the front surface, that is why the front surface is bright and sharp in both microscope images. Another portion of the rays leaves the chip at the sides and illuminates the whole area of the back reflector. After reflecting and scattering on it they leave the package mainly in skew directions. The optical model of the LED and two measured microscope images justify that both the chip and the internal reflector of the LED “radiate”, thus both of them should be taken into account when designing light sources with this LED. A good practical estimation is that the total emission of this LED comes from a circular aperture having the same diameter as the aperture of the back reflector (1 mm). It is important to emphasize that this 1 mm aperture diameter is much less than the diameter of the plastic package (3.25 mm) but larger than the lateral size of the chip (0.4*0.4 mm).
4.3.2 The calculation of the optimal parabolic shape and aperture

The basic setup of our light source consists of an LED, a cylindrical parabolic mirror and a blocking aperture (see 4.9. Fig.). The blocking aperture, which has a rectangular shape, serves to stop the rays propagating "far" from the optical axis and so reduces the width of the stripe. At the same time it reduces the total emitted power by blocking a portion of the emission.

The design evaluation consisted of the following steps: I tested the basic setup with parabolic cross sections of different sizes ( \( R = 60, 70, 80, 90 \) mm, parabola equation: \( y = \frac{1}{2R}x^2 \) [mm] ) and with different aperture heights (from 20 to 90 mm). I calculated the resulting stripe widths at 2m distance and the ratios of the power collected into the stripe over the total emitted light power. These values were estimated as the ratio of the number of rays transmitted by the aperture relative to the total numbers of rays that arrived at the parabolic mirror. This is an important characteristic for the different setups, because apertures of different heights transmit different percentages of the total radiation. With the parabolas of 60 and 70 mm radii the divergence angle of the illuminated light was too high and this resulted too high stripe widths at reasonable light collection efficiencies. The higher divergence angle of the illumination is caused by the shorter focal lengths of the parabolas of shorter radii. 4.1. Table and 4.2. Table show the calculation results for \( R = 80 \) and 90 mm parabolic cross sections, which make the two best candidates for choosing the most effective parabolic shape for the stripe illuminator.
<table>
<thead>
<tr>
<th>Aperture height [mm]</th>
<th>Calculated stripe width at 2m distance form the mirror [mm]</th>
<th>Calculated light collection efficiency [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
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</tr>
<tr>
<td>20</td>
<td>30</td>
<td>23</td>
</tr>
</tbody>
</table>

Results of calculations for the cylindrical mirror of parabolic cross section defined by \( y = \frac{1}{(2 \times 80\text{mm})} x^2 \) [mm]

4.1. Table

<table>
<thead>
<tr>
<th>Aperture height [mm]</th>
<th>Calculated stripe width at 2m distance form the mirror [mm]</th>
<th>Calculated light collection efficiency [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>90</td>
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<td>44</td>
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<tr>
<td>30</td>
<td>29</td>
<td>33</td>
</tr>
</tbody>
</table>

Results of calculations for the cylindrical mirror of parabolic cross section defined by \( y = \frac{1}{(2 \times 90\text{mm})} x^2 \) [mm]

4.2. Table

All the stripe widths shown in 4.1. Table and 4.2. Table are under 60mm, but I supposed that the stripe width of the real source would become bigger because of the misplacements and bad alignments of the LEDs. Therefore the R=80 mm parabolic cross section with 60 cm aperture height (see the highlighted row in 4.1. Table) was selected as an optimal setup for the application. 4.10. Fig. shows the projection of the ray paths in the yz-plane for this setup. According to the calculations in ideal case this setup can supply a light stripe of 4 cm width at 2 m distance that is below the required 6-10 cm. It is worth to mention that the selected cylindrical parabolic mirror has a focal length of 40 mm.

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**Projection of ray paths to the yz-plane for the cylindrical mirror with a parabolic cross section, described by \( y = \frac{1}{(2 \times 80\text{mm})} x^2 \) [mm], and with 60 cm aperture height**

4.10. Fig.
4.3.3 Tolerance analysis

Definition of misplacement parameters for tolerance analysis

All previously described calculations were made with one LED and the LED was in the optimal position and orientation in front of the cylindrical-parabolic mirror. In the realisation a linear array of LEDs is used that are mounted onto a printed circuit stripe. In this case the misplacements of the different LEDs cause an increase of the stripe width. Therefore a tolerance analysis was carried out considering the errors in the position and orientation of the LEDs. The position of any misplaced LED can be expressed with 3 misplacement vectors (DX, DY, DZ). The misplacement DX of the LEDs has no significance, because the x axis is parallel with the focal line of the parabolic mirror.

The orientation of a misplaced LED can be expressed with three rotation angles (TLA, TLB,TLC). The meaning of these misplacement parameters is shown in 4.11. Fig. In the calculations each LED is considered as a rotationally symmetric device ignoring that the emitting chip has a rectangular shape. Originally the axis of the LED is parallel with the z axis, then the LED-axis is first rotated around the x axis by TLA, and then rotated by TLB around the already rotated y axis. TLC means the rotation of the LED around the already twice (TLA, TLB) rotated z axis that is identical with the symmetry axis of the LED. Because of the assumed rotational symmetry of the LED this last rotation (TLC) has no influence. The results are summarised in 4.3. Table. The stripe-widths given in 4.3. Table are calculated as the maximal differences between the ray with maximal y coordinate and the ray with minimal y coordinate incident at 2m distance from the mirror. This table shows that for keeping the stripe width under 10 cm, which was the goal of our design, a very accurate mounting is necessary (see the highlighted row).
The position of the chip in the package varies with an uncertainty of 0.5 mm (catalog data) for the LED type used, which is larger than the required 0.2 mm positioning accuracy. Therefore sorting of the LEDs is necessary by measuring the chip position compared to the surfaces of the package. The position of the chip relative to the cylindrical side of the package (in x and y directions) was measured with an Abbe comparator of 1µm resolution. The position of the front surface of the chip relative to the front surface of the package (in z direction) was measured by an optical microscope and a dial meter of 10µm resolution. The microscope was focused first onto the chip surface and second onto the front surface of the package. The movement of the plate holding the LED was measured between the two positions by the dial meter that allowed to calculate the needed distance.
4.4 Realisation

Production of non-rotational symmetric optical elements is in general difficult. The cylindrical parabolic mirror was realised in the following cost effective way: metal ribs were formed to the required parabolic shape by a high precision laser cutting machine, 5 ribs like these were mounted side by side into a lamp-house, and a flexible reflecting foil was stretched on the ribs by springs. By stretching the mirror foil onto the ribs it took the parabolic shape required. The linear LED array consisted of 72 elements. First, by sorting the LEDs, 72 pieces were selected. The position difference of the chip as compared to the package was less than 0.1mm for the selected LEDs. Since by simple soldering onto a printed circuit stripe neither the 0.2mm position accuracy, nor the 0.3° angular accuracy could be reached, a special mounting frame was constructed for the precise positioning of the LEDs before soldering, concerning y and z positions and TLA and TLB directions. After soldering all the LEDs, they were removed from the frame. A photo of the LED array can be seen in 4.12. Fig., and the whole light source is shown in 4.13. Fig..
4.5 Results and conclusions

4.5.1 Optical and geometrical characteristics

4.14. Fig. shows the angular distribution of the irradiance in the xz plane. The irradiance values were measured at the central plane of the stripe at 1 m distance from the middle of the source. In the area subtending a plane angle less than ±60° with the z axis the irradiance is over the 80% of the maximum, which means that the source illuminates here homogenous light. In the angular areas over ±60° the irradiance decreases fast, and reaches 10% of the maximum at ±75°. The 50% of the maximal irradiance is about at ±65°. The cause of this very fast irradiance decreasing over ±60° is that the sides of the lamp house block the rays propagating at large angles with the z axis. These considerations show that the obstacles lying inside of the central angular area of ±65° can be effectively illuminated and measured by one source like this. By the application of more pieces of differently orientated sources and by improving the construction of the lamp house the illuminated angular area can be extended.
4.15. Fig. shows the distribution of the irradiance along the direction normal to the stripe (y direction). This measurement was done at 2m distance in the z direction from the middle of the source. As presented in 4.15. Fig. the light source irradiates a 75mm wide central zone with irradiiances more than 50% of the maximum. This is the central and most intense zone of the light source. The next zone of the stripe extends up to a width of 130mm, which receives irradiance of 10-50% of the maximum. The last zone of the stripe is between 130 and 200mm widths receiving 3-10% of the maximal irradiance. Between 0 and 65mm half widths (upto 130mm total width) the gradient of the irradiance is much higher than between 65 and 100mm half widths. The probable cause of that is that most of the LEDs were mounted with such tolerances that they emit all of their radiation into this 130mm wide zone. The irradiance over the 130mm total width is probably caused by the more inaccurate mounting of a small number of the LEDs and the light scattering on the surfaces with poor optical quality.

If we accept that most of the LEDs emit into a 130mm wide zone, we get that the achieved mounting accuracies of these LEDs were: DCY=±0.3mm, DCZ=±0.3mm, TLA=±0.5°, TLB=±0.5° (see 4.3. Table).

During the localizations of objects that have a y size larger than the stripe width and therefore become illuminated by the total stripe width, the signal caused by the central zone dominates, and the signal coming from the outer regions of the stripe do not play any role. For this kind of objects the central zone (75mm wide) constitutes the stripe width. The objects in the environment of a mobile robot (walls, furniture, doors, people,...) are mostly of this type. The objects that have a y size much smaller than the stripe width (y size of object << 13cm) are illuminated only by a small part of the total stripe. So for them a stripe width of 150 or 200 mm can be estimated. Such small objects are found rarely in the environment of a mobile robot.
### Table

<table>
<thead>
<tr>
<th>Distance [cm]</th>
<th>Irradiance [mW/cm²]</th>
<th>Spectral Irradiance [µW/cm²/nm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>5.5</td>
<td>69</td>
</tr>
<tr>
<td>100</td>
<td>1.1</td>
<td>14</td>
</tr>
<tr>
<td>200</td>
<td>0.6</td>
<td>7.5</td>
</tr>
</tbody>
</table>

4.4. Table summarises the irradiances of the stripe at different distances from the light source. The irradiances were measured at the maximal point of the central zone as the peak values by pulsing the LEDs with 200mA forward current. An important characteristic of a stripe illuminator is, as it was discussed in the introduction, the ratio between the spectral irradiances caused by the light source and by the background illumination. According to our measurements, in indoor environment the spectral irradiance of the background illumination varies between 0.1 and 2 µW/cm²/nm in the wavelength range of 800-1100nm, which means that the signal to noise ratio calculated for the value of our light source measured at 1m distance (14µW/cm²/nm) varies between 140 and 7. These SNRs show that the light source can be very effectively applied under indoor illuminating conditions.

In outdoor environments stronger background illumination can be present, as in the case of bright sunshine. According to the Infrared Handbook [12.] the sun causes a spectral irradiance of 86µW/cm²/nm measured at sea level with normal irradiance at 850nm wavelength. Comparing this value with the 14µW/cm²/nm spectral irradiance of our light source we signal-to-noise ratio of 0.16 results. The signal-to-noise ratio can be enhanced by using special filtering and integrating methods as e.g box-car or lock-in detection.

It is important to investigate the spectral emission and spectral response functions of some physical components that have impact on the SNR achievable with the stripe illuminator. 4.16. Fig./a shows the spectral emission of the Siemens SFH-487 LED. The LED emits in a narrow spectral band of approx. 75nm FWHM with an emission peak at 880nm. It can be accepted that the whole light source has the same spectral emission as a single LED provided the variations of the spectral emissions of the different LEDs are not taken into account. 4.16. Fig./b shows the spectral response of a typical silicon CCD. Stripe light sensors apply usually such CCDs as image detectors. The curve shows that the silicon CCD is sensitive in the visible and infrared range upto 1050nm. The background illumination of the measured environment has also a strong impact on the SNR achievable with the stripe illuminator. Indoor rooms are usually illuminated by incandescent lamps, fluorescent lamps and by the sun. The spectral emission of a tungsten incandescent source of 3200K is shown in 4.17. Fig and the spectral emission of the sun is shown in 5.9. Fig. Both curves show that incandescent sources illuminate in the whole visible and infrared wavelength regions. In order to reduce the noise resulting from this wide band background illumination an optical bandpass filter fitted to the spectral emission of the LEDs should be applied in the signal detecting optics. Above in this point the SNR values were defined as the ratio of the spectral sensitivities of the illuminated stripe and of the background light source.
light. This SNR values can be only achieved or even approached if a proper optical bandpass filter is applied to suppress the impact of the background illumination outside of the emission band of the LEDs. If such an optical bandpass filter is not applied, then much smaller SNRs can be expected.

4.5.2 Eye Safety

There are two possible ways to analyse the eye safety of the light source. One way is to examine the eye safety of the source using the principles of optics and the published material about eye safety. The other way is the classification of the light source according to valid the eye safety standards. As part of the main text I will examine the eye safety only on the former way since only the knowledge of optics is required for the understanding in this case. An eye safety analysis according to the valid
standards, which is unavoidable for the practical applications, is presented in Appendix I. This analysis is given in the appendix since it does not represent valuable information for readers not familiar with the standards. For achieving the results of this point I use the chapter 4.5 of the basic work of Sliney and Wolbarsh about the eye safety of lasers and other optical sources [13.].

A simple schematic drawing about the structure of the human eye is shown in 4.18. Fig. The two imaging elements of the eye (cornea and lens) create a real image of the objects on the retina. The retina consists of very complex layers of nerve cells and it is actually an extension of the brain. The image is detected by cones and rods, cones are sensitive for colours and rods are only sensitive for brightness. The light source illuminates in a narrow spectral band centered at 880 nm, which is transmitted by both the cornea and lens. The main eye injury hazard is the retinal injury due to the high temperature rise induced by the sharp image of an object with high brightness. As it will be shown later the cornea, aqueous and lens absorb a significant portion of the incident radiation and a too high dose of absorbed light could make them loose their transparencies because of the induced photochemical effects. Extremely high intensity illumination can also damage the iris however in [13.] no quantitative data is published about this type of hazard.
4.5.2.1 Retinal hazard

The cylindrical parabolic mirror creates two distinct virtual quasi-images about each LEDs provided the LEDs are placed on the focal line. The reason of having two distinct virtual quasi-images is that for rays propagating in the yz plane the mirror works as parabolic mirror with finite focal length, while for rays propagating in the xz plane the mirror acts as a planar mirror with infinite focal length. The presence of two different focal lengths for rays in the yz and in the xz planes leads to two distinct quasi virtual images.

Let’s examine the virtual images created by the ray components parallel with the xz plane at first. In this case the quasi-virtual images are distinct rectangles with an x-size equal to the x-size of the LED and an y-size approximately equal to the aperture size of the parabolic mirror in y direction. In reality these rectangular areas are curved in the yz plane and take parabolic forms, but I neglect to take these curved shapes into account, namely I use the paraxial approximation of the mirror here. Since the LEDs are placed into the focal line of the mirror the distance of the virtual images from the mirror is equal to the focal length (see 4.20. Fig.). For the analysis of eye injury a “chip LED approximation” is applied assuming that the total light power is emitted through the front surface of the LED chip (0.4x0.4 mm). This approximation reduces the size of the real LED, consequently increases the brightness of the source and the hazard of retinal injury. The surface of the LED chip is assumed to be a spatially homogenous Lambertian radiator, thus the angular distribution of its radiance is constant.
4.20. Fig. shows yz view and 4.21. Fig. shows the xz view of the light source, the virtual image and the observer’s eye. I assume that the observer looks into the light source at the closest possible distance, namely he places his eye into the plane of the LED array (see 4.20. Fig.). As the cylindrical parabolic mirror collimates the radiation in y-direction, all points of the virtual image of an LED radiate with a divergence angle of \( \Delta \alpha = \frac{d \ell}{f} \) measured in the plane yz (see 4.20. Fig.). Since \( \Delta \alpha \) is small, a single point of the virtual image radiates only into a small part of the of the eye’s aperture in y-direction, but not into the whole y-size (see 4.20. Fig.). In direction x the angular distribution of the LED’s radiation is not modified by the cylindrical parabolic mirror so all points of the virtual image emit into the whole x-size of the eye’s aperture (see 4.21. Fig.). According to these considerations a point of the virtual image radiates only into a narrow stripe of the eye aperture as shown in 4.22. Fig.
The highest retinal hazard takes place if the eye is focused onto the plane of the virtual images of the LEDs, namely when the LED chips are imaged into distinct rectangular areas of the retina. In order to evaluate the hazard of retinal injury the irradiance absorbed by the retina should be calculated. The parameters used in the calculation are defined below.

- $f=40$ mm: focal length of the parabolic mirror
- $D$: aperture height of the parabolic mirror
- $f_e=13.7$ mm: focal length of the eye
- $d_e=7$ mm: diameter of the eye (iris)
- $d_l=0.4$ mm edge length of the LED chip
- $R$: radiance of both LED chip surface and virtual image

The solid angle $d\Omega$ covered by the narrow stripe of the eye aperture in which the infinitesimal surface $dA$ illuminates can be calculated by Eq. (4.1).

$$d\Omega = \frac{\Delta\alpha \cdot 2 \cdot f \cdot d_e}{4 \cdot f^2} = \frac{d_l \cdot 2 \cdot f \cdot d_e}{4 \cdot f^2} = \frac{d_l \cdot d_e}{2 \cdot f^2}$$  

4.1. Eq.

The light power that is illuminated by the infinitesimal surface $dA$ of the virtual source into the eye aperture can be calculated on the following way:

$$dP = R \cdot dA \cdot d\Omega = R \cdot dA \cdot \frac{d_l \cdot d_e}{2 \cdot f^2}$$  

4.2. Eq.

I assume that the eye focuses onto infinity, namely the distance between the retina and the eye is equal to the focal length of the eye ($f_e$). In this case the surface element $dA$ of the virtual source is imaged onto the retina with a linear magnification of $m=f_e/2f$ (see 4.20. Fig.). So the irradiance absorbed by the retina can be calculated with the following equation:
The new parameter $\tau$ denotes the relative spectral effectiveness of retinal thermal injury including the transmission of the ocular media (cornea, aqueous, lens, vitreous) and the reflection and absorption of the retina. $\tau$ is approximately equal to 0.15 at 880 nm wavelength according to Fig. 4-12 in [13]. The radiance of the LED chip surface can be calculated on the following way assuming that the total light is emitted by the front surface of the chip which is a spatially homogenous Lambertian radiator. For a Lambertian radiator the relation between the total emitted power and the radiance is $P=R^*\pi^*A^*$, where $A$ denotes the surface of the radiator. The light source is planned for impulse operation with 1/25 s impulse time and 5 Hz repetition frequency. For this type of operation the LEDs can emit 40 mW total light power, so the radiance of the LED chip is $R=40\text{mW}/\pi/0.16\text{mm}^2=80\text{mW}/\text{sr}/\text{mm}^2$. Since now all the parameters of 4.3. Eq. are known, the irradiance absorbed by the retina can be calculated as follows:

$$I_r = 2 \cdot R \cdot \frac{dl \cdot de}{fe^2} \cdot \tau$$

4.3. Eq.

The Maximal Permissible Exposure (MPE) of the retina for 0.15 s light impulses originated from extended sources is $1\text{W/cm}^2=10\text{mW/mm}^2$ according to Figure 4-20 of [13]. As the calculations show the MPE for an impulse of 0.15 s duration time is 27 times higher than the absorbed retinal irradiance caused by our source.

---

* The total power emitted by a Lambertian radiator of radiance $R$ and surface size $A$ into a $2\pi$ sr hemisphere can be calculated by the following integral:

$$P = \int_0^{\pi/2} R \cdot A \cdot \cos(\theta) \cdot 2 \cdot \sin(\theta) \cdot \pi \cdot d\theta = R \cdot A \cdot \pi \cdot \left[ -\cos(2 \cdot \theta) \right]_0^{\pi/2} = R \cdot A \cdot \pi$$
As it was discussed before, different virtual quasi images of each LED are created if the rays propagating parallel with the yz plane are taken into account. Regarding these rays, the mirror acts as a parabolic mirror and so each LED, which are placed in the focal line of the mirror, is “imaged” virtually into a stripe parallel with axis x at infinite distance (practically far enough) behind the mirror (see 4.23. Fig.). The different elements of the LED array are imaged virtually into different stripes that are shifted in direction x relative to each other by the same amount as the LEDs are separated relative to each other within the array. Summing up the irradiances of the individual stripes, we get a stripe formed irradiance distribution representing the virtual image of the whole LED array. I would like to emphasize that the stripe is continuously illuminated and all the LEDs illuminate into each point of it. If the observer focuses his eye onto infinity, then this stripe will be imaged into a line of the retina.

Let’s take a surface element dA of the virtual image at infinite distance lying on the optical axis of the eye as 4.23. Fig. shows. In the yz-plane, the surface element dA illuminates into the whole aperture of the parabolic mirror, consequently dA illuminates into the whole aperture of the eye in the yz-plane (see 4.23. Fig./a). In the xz plane (see 4.23. Fig./b) the surface element dA illuminates only in discrete angular sections defined by the virtual images of the LED chips at distance f behind the lens. Within
such a section the radiance is equal to the radiance of the LED chip surface. In order to calculate the average radiance of the virtual image the source built of an array of discrete sources can be theoretically replaced by a homogenous emitter having a radiance equal to the average radiance of the LED chip array. Such a continuous source would illuminate the same total power homogenously distributed in the whole angular space in the xz-plane. The radiance of this homogenous emitter can be calculated by multiplying the radiance of the LED chip with the linear duty factor of the LED chips within the array, namely \( \text{Rh}=80 \text{mW/sr/mm}^2 \times 0.4 \text{mm}/3.5 \text{mm} = 9.14 \text{mW/sr/mm}^2 \). As the virtual image at infinite distance of such a homogenous rectangular emitter radiates into the whole aperture of the eye, the light power radiated by the surface element \( dA \) can be calculated with 4.4. Eq., in which \( s \) denotes the distance between surface \( dA \) and the eye.

\[
dP = \text{Rh} \cdot dA \cdot d\Omega = \text{Rh} \cdot dA \cdot \frac{d^2 \cdot \pi}{4 \cdot s^2}
\]

4.4. Eq.

The irradiance absorbed by the retina can be obtained by using the linear magnification of \( m=\frac{fe}{s} \) between surface element \( dA \) and its retinal image as follows:

\[
\text{Ir} = \text{Rh} \cdot dA \cdot \frac{d^2 \cdot \pi}{4 \cdot s^2} \cdot \frac{s^2}{\text{dA} \cdot \text{fe}^2} = \text{Rh} \cdot \frac{d^2 \cdot \pi}{4 \cdot \text{fe}^2} \cdot \tau
\]

4.5. Eq.

\( \tau \) denotes here the spectral effectiveness of retinal thermal injury as in the case of 4.3. Eq. Applying 4.5. Eq. the irradiance absorbed by the retina can be calculated on the following way:

\[
\text{Ir} = 9.14 \frac{\text{mW}}{\text{mm}^2 \cdot \text{sr}} \cdot 49 \text{mm}^2 \cdot \frac{\pi \text{sr}}{4 \cdot 13.7^2 \text{mm}^2} \cdot 0.15 = 0.28 \frac{\text{mW}}{\text{mm}^2}
\]

As the Maximal Permissible Exposure (MPE) for a single pulse of 0.15s is 1W/cm²=10mW/mm², so in this case the absorbed irradiance is 36 times smaller than the MPE. Consequently the physical calculations show that a single pulse of the light source is eye safe for both virtual images however for practical applications an analysis according to the eye safety standards should be made. These physical calculations were necessary as the standards are changing from time to time. Another conclusion of this analysis is that the former virtual image at distance \( f \) behind the lens is more dangerous than the latter one created at infinity, so the former case will be used in the eye safety analysis according to the standard. Unfortunately the MPE for pulse trains is not explicitly published in [13].

### 4.5.2.2 Infrared cataract

The second type of eye injury caused by IR illumination is cataract formation on the eye lens by which the lens becomes opaque. Cataract formation can be caused by high power exposure of short term and small power exposure of long term. The cause of cataract formation for short term and high power impulses is the heat transferred by the iris to the lens. For this type of cataract formation very
intense corneal exposure is needed (100W/cm²) that is much higher than the exposure of the source (see 4.4. Table). The cataract genesis because of long term exposure is caused by the photochemical reactions within the eye lens induced by the absorbed IR illumination. The long term means here from several years up to 10-20 years. There is no clear eye safety limit against long term infrared cataracts according to point 4.6.2 of [13.], but it is stated that “safe chronic ocular exposure values, particularly to IR-A, are of the order of 10 mW/cm² or below.”. As 4.4. Table shows the maximal corneal exposure caused by the source is 5.5 mW/cm² for a short impulse time of 1/25 sec. The average exposure of the cornea at 5 Hz impulse operation is 1.1 mW/cm² and this value should be considered for long term cataract formation. According to the data published in [13.] the light source is surely safe against short term and likely safe against long term cataract genesis in the eye lens. Infrared cataract can be also created on the cornea by IR-B (1400-3000 nm) illumination, but IR-A (760-1400 nm) is not dangerous since the cornea is totally transparent in this spectral region.

4.5.3 Results on a mobile robot

I built the light source into a sensor system [14.] and demonstrated its operation on a mobile robot. The mobile robot used was ROAMER II [15.][16.] without its sonar sensing system. The sensor could detect different kinds of obstacles except glass objects with finely polished transparent or reflective surfaces. The resolution of the sensor varied between 5-15cm depending on the measured range. In inside environments, the ROAMER II could fulfil all of its navigation tasks without colliding with the obstacles around it by relying only on data about the environment obtained from the sensor. These experiments demonstrate the efficient operation of the light source in a stripe-lighting sensor on a mobile robot under indoor lighting circumstances.
5 SNR analysis of the stripe illuminator based on linear emitter and cylindrical lens against solar irradiance

5.1 Introduction

Mobile robots require different types of sensors in indoor and outdoor applications, since the disturbing effects in outdoor environments are stronger and originating from more sources than in indoor environment. An important disturbing effect is the very strong background illumination in an outdoor environment because of direct sunshine. To overcome this problem 2D or 3D laser scanners are often applied [17.]. By focusing the laser into a single spot the solar radiation can be spectrally over illuminated and the analog measurement of the back-scattered signal allows the application of effective filtering methods, such as temporal modulation or lock-in detection. The measurement principle of such sensors is usually time-of-flight measurement. The main drawback of these sensors is their high price because of the applied high frequency measuring electronics and precision scanning mechanisms. Ultrasonic sensors are not disturbed by the background illumination, but they have limitations in accuracy, reliability and measuring distance. Another possibility is the application of stripe lighting sensors because of their inherently easy construction (stripe source, camera, computer), good reliability and good accuracy. Stripe lighting sensors usually apply image detector arrays (CCD or CMOS detectors), which prevents the signal enhancement by analog electronic filtering techniques, thus only digital filtering methods can be applied.

Several stripe-lighting sensors have been published [2.][3.][5.][18.][19.] in the field of robotics, which work properly in indoor illuminating conditions, but their outdoor behaviors have not been analyzed. Röning and Haverinen published a stripe lighting sensor for outdoor robotic applications [1.], which uses a 50mW laser diode at 780nm wavelength in the light source and a matching optical band pass filter of 20nm bandwidth in the imaging system. The sensor worked effectively in such outdoor environments, where no direct sunshine was present. To distinguish the signal from the noise originating from background illumination special image processing filters, such as intensity profile recognition were used in addition to the conventional temporal light source modulation, but the high noise caused by direct sunshine could not be overcome. Viitanen et. al. present an integrated mobile robot control system for outdoor container handling [20.]. The system integrates the signals of odometric, ultrasonic, stripe-lighting, depth from focus and GPS measuring systems to resolve the very complex challenges of outdoor environments, as high external illumination, rain, snow, fog, vehicle vibration because of uneven ground, etc. The applied stripe-lighting sensor is probably the same as the one published in [1.]. The experience of the authors was the same, namely that the stripe lighting sensor did not give reliable signal for objects directly illuminated by sunshine. The stripe
illuminator based on a one dimensional LED array and a cylindrical parabolic mirror presented in chapter 4 worked very effectively in indoor illuminating conditions but could not be applied in outdoor environment.

The common conclusion of the bibliography research and our previous work is that improving the signal to noise ratio against solar irradiation by applying appropriate light source constructions is an essential requirement for stripe lighting sensors operating in outdoor environment. Papers examining this problem have not been found in the scientific literature. This chapter is devoted to the problem of finding stripe illuminators supplying good signal to noise ratios (SNR) against solar illumination. The SNR is defined as the ratio of the spectral irradiance caused by the source and the spectral irradiance of sunshine, as it will be discussed in this paper with more details. The chapter focuses onto the stripe illuminator constructed of a cylindrical lens and a rectangular emitter placed at the focal line of the lens (5.1. Fig.), which is the generalized form of the light source presented in chapter 4. The main advantage of this source type is that it can supply very high total light power because of the large surface size of the emitting rectangle. High light power is an important requirement for outdoor applications. Another positive feature is the increased stripe homogeneity compared to sources using a single emitter of small size.

First the radiometric analysis of the source is accomplished. The result of the analysis is an integral by which the spatial irradiance distribution within the stripe can be calculated. Designers can effectively apply this integral for calculating the spatial irradiance distribution of stripe sources based on the analyzed concept. Experimental stripe illuminators applying different types of real emitters, such as LED array, laser diode array and incandescent filament are theoretically analyzed. The SNRs against
sunshine are calculated by numerical evaluation of the achieved radiometric integral for the experimental sources.

Throughout this chapter the following radiometric terminology is applied [21.]: the surface density of radiant power is called irradiance (W/m²); the solid angle density of radiant power is called radiant intensity (W/sr); the surface and solid angle density of radiant power is called radiance (W/m²/sr).

5.2 The radiometric analysis of the stripe illuminator

5.2.1 Definition of source parameters

An effective high power stripe sources can be built of a cylindrical lens and a rectangular emitting source placed on the focal line of the lens (5.1. Fig.). The x-size of the emitter is much smaller than the y-size. In 5.2. Fig. the co-ordinate system used later in this chapter and the following parameters of the light source are defined:

- f: focal length of the cylindrical lens
- D: aperture size of the cylindrical lens in x direction
- L: aperture size of the cylindrical lens and size of the rectangular source in y direction
- w: size of the rectangular source in x direction
- d: distance of the xy detection plane from the lens

In points 2.2 and 2.3 a radiometric integral (5.12. Eq.) is created by which the irradiance can be calculated in any point of the illuminated stripe. This integral assumes the knowledge of the geometrical parameters of the stripe illuminator (emitter and lens), and the angular radiance distribution of the rectangular emitter. Those readers, who are not interested in the details of the radiometric analysis, might omit points 2.2 and 2.3.
5.2.2 Radiance distribution at the exit aperture of the cylindrical lens

Our radiometric model assumes a spatially homogeneous rectangular emitter, namely the angular distribution of the emitter’s radiance is the same in all points of the source. Our considerations are valid only for spatially incoherent emitters because of the applied radiometric means. So in our system a spatially homogenous and incoherent primary source illuminates the cylindrical lens, by which the illumination is transformed. Therefore the exit aperture of the cylindrical lens can be regarded as a secondary source. In this point the radiance distribution of this secondary source is calculated. The exact radiometric modeling of a real cylindrical lens is difficult. Therefore an idealized model is applied assuming that the cylindrical lens images the rectangular source into a stripe. Furthermore the cylindrical lens is assumed to have zero thickness. As it is shown later, these assumptions lead to the violation of radiation conservation theorem [21.] in the case of a high numerical aperture lens. Here I refer to the fact that the model of a rotational symmetric ideal thin lens also violates this theorem.

\[ \Delta \phi(\beta) = \frac{w}{f} \cdot \cos(\beta) \]

5.1. Eq.

All points of the lens exit aperture radiate with \( \Delta \phi(\beta) \) divergence angle in the plane x-\( \beta \). 5.1. Eq. is equivalent to the assumption that the xy image of the rectangular source is a stripe. The stripe width in
x direction can be calculated with the following formula in a detection plane \( xy \) located at a distance \( d \) from the lens using 5.1. Eq.(5.3. Fig.):

\[
S(\beta, d) = D + \frac{w}{f} \cdot d
\]

5.2. Eq.

\( S(\beta, d) \): stripe width
\( d \): distance of the detection plane \( xy \) from the lens

Since our ideally thin cylindrical lens has zero thickness, the lens transforms the radiation only angularly but not spatially. We discuss now how the radiation is angularly transformed by passing through the lens and the spatial and angular distribution of the radiance after the lens is also calculated.

\[90 \text{ deg}\]

**Definition of parameters for calculating the radiation transfer between elementary surfaces of the emitter and the cylindrical lens**

5.4. Fig.
The parameters used in 5.4. Fig. and 5.5. Fig. are the following:

\( \alpha \): plane angle subtended by the beam propagation direction and the normal vector of the lens aperture (vector \( z \))

\( \beta \): plane angle subtended by the projection of the beam propagation direction onto plane \( yz \) and the normal vector of the lens aperture (vector \( z \))

\( dl \): length of infinitesimal source in direction \( y \)

\( dA \): surface size of infinitesimal surface of the lens aperture

\( xa \): \( x \) coordinate of infinitesimal surface of the lens aperture

\( P \): center of the infinitesimal surface \( dA \)

\( R(\alpha, \beta) \): Radiance (W/m\(^2\)/sr) of the emitter in the beam propagation direction defined by \( \alpha \) and \( \beta \)

The light power that the infinitesimal emitter \( w \cdot dl \) illuminates into the infinitesimal surface \( dA \) of the lens aperture (5.4. Fig.) can be calculated by 5.3. Eq. using the power transfer equation of radiometry \([21.]:\)

\[
\text{d}P = R(\alpha, \beta) \cdot \frac{w \cdot dl \cdot dA \cdot \cos^4(\alpha)}{f^2}
\]

The lens radiates this light power into a modified solid angle. Let’s start four single rays from the four vertices of the infinitesimal emitter toward the center of \( dA \) (5.5. Fig.). As 5.5. Fig./a shows the four rays, after passing through point \( P \), form a pyramid with rectangular base. All the radiation that stems from any point of the elementary source and passes through \( P \) is radiated inside this pyramid. Therefore point \( P \) radiates into the solid angle, covered by this pyramid. 5.5. Fig./b shows the projection of the ray paths onto the planes \( x-\beta \) and \( y-z \). The divergence angle of the rays measured in
The plane $x-\beta$ can be calculated by 5.1. Eq. The divergence angle measured in the plane $y-z$ is (5.5. Fig./b and 5.4. Fig.) $\Delta \delta = \frac{dl}{f} \cdot \cos^2(\beta)$. So the plane angles between the opposite side-surfaces of the pyramid are $\Delta \phi$ and $\Delta \beta$ (5.5. Fig./a). Such a pyramid covers the solid angle (using 5.1. Eq.)

$$\Delta \Omega_s = \Delta \phi \cdot \Delta \beta = \frac{w \cdot dl}{f^2} \cdot \cos^3(\beta)$$

5.4. Eq.

The radiation leaves the lens at an angle $\beta$ to the surface normal of the lens aperture (5.3. Fig.). Using 5.3. Eq. and 5.4. Eq. the radiance distribution of the light leaving the cylindrical lens can be calculated by 5.5. Eq. in any point of the exit aperture.

$$\frac{dP}{dA \cdot \cos(\beta) \cdot \Delta \Omega_s} = R(\alpha, \beta) \frac{\cos^4(\alpha)}{\cos^4(\beta)}$$

5.5. Eq.

The angle $\alpha$ can be calculated from $x_a$ and $\beta$ with the following trigonometric equation (5.4. Fig.): $\alpha = \frac{\tan^{-1}\left(\sqrt{x_a^2 + f^2 \cdot \tan^2(\beta)}\right)}{f}$. Substituting this result into 5.5. Eq. yields:

$$\text{Rs}(x_a, \beta) = R(\alpha, \beta) \cos^4\left(\tan^{-1}\left(\frac{\sqrt{x_a^2 + f^2 \cdot \tan^2(\beta)}}{f}\right)\right) \cdot \cos^4(\beta)$$

5.6. Eq.

5.6. Eq. gives the radiance distribution of the light leaving the lens at any point of the lens aperture. All points of the aperture, having an $x$ coordinate of $x_a$, radiates with $\text{Rs}(x_a, \beta)$ radiance parallel with plane $x-\beta$. This radiance is emitted only within the plane angle of $\Delta \phi(\beta)$ in plane $x-\beta$ (5.5. Fig./b). The radiance in plane $x-\beta$ outside the plane angle $\Delta \phi(\beta)$ is zero. According to 5.6. Eq. the radiance is independent on $y$, which can be explained by the invariance of the source’s construction for translation along axis $y$ (not taking the side effects into account).

As it was already mentioned 5.5. Eq. and thus also 5.6. Eq. violate the radiation conservation theorem.

But for a cylindrical lens with small numerical aperture ($\text{NA}=D/2f$) the approximation of $\cos^4(\alpha) = \cos^4(\beta)$ can be applied, so the radiation conservation theorem is not violated in this case. 5.5. Eq. shows that the higher the inequality between $\alpha$ and $\beta$ is, the higher the inequality between the incident and the leaving radiances becomes. It supports just the well known fact that idealized modeling is inaccurate for high numerical aperture lenses.
5.2.3 Calculation of the irradiance behind the lens

Using 5.6. Eq. the irradiance behind the lens is calculated at any x-y detection plane in this point. Any point of the lens aperture radiates a divergent radiation with a divergence angle of $\Delta \phi (\beta)$ parallel with plane x-$\beta$. This divergent radiation illuminates a stripe of the x-y detection plane that has an x size given by 5.2. Eq. Let’s take an observation point O inside the illuminated stripe (5.6. Fig.). Point O can receive radiation with divergence angles of $\Delta \phi (\beta)$ in the different x-$\beta$ planes including point O (5.6. Fig.). So the x size of the section of the lens aperture that illuminates into O in any x-$\beta$ plane can be calculated with 5.7. Eq. using 5.1. Eq.

$$h(\beta) = \Delta \phi (\beta) \cdot \frac{d}{\cos(\beta)} = \frac{w}{f} \cdot d$$

5.7. Eq.

As 5.7. Eq. shows $h(\beta)$ does not depend on $\beta$, thus point O receives radiation from a rectangular section of the lens aperture with $w/f \cdot d$ x-size. This is true only if the two ends of $h(\beta)$ do not expand over the borders of the lens aperture. Let’s denote the x coordinates of point O and the two ends of $h(\beta)$ with $x_0$, $x_1$ and $x_2$ respectively, as shown in 5.6. Fig.. The parameters $x_1$ and $x_2$ can be calculated by 5.8. Eq./a and 5.8. Eq./b.

$$x_1 = x_0 - \frac{w}{2 \cdot f} \cdot d \quad \text{if} \quad x_0 - \frac{w}{2 \cdot f} \cdot d \geq -\frac{D}{2}$$

$$x_1 = -\frac{D}{2} \quad \text{otherwise}$$

(a)
\[ x_2 = x_0 + \frac{w}{2 \cdot f} \cdot d \quad \text{if} \quad x_0 + \frac{w}{2 \cdot f} \cdot d \leq \frac{D}{2} \]

\[ x_2 = \frac{D}{2} \quad \text{otherwise} \]

5.8. Eq.

---

**Section of cylindrical lens aperture that radiates into an observation point**

5.7. Fig.

---

**Parameter definition for calculating the irradiance, caused by an elementary surface of the cylindrical lens in an observation point**

5.8. Fig.

In direction \( y \) the radiation of the whole aperture reaches point \( O \). Instead of \( y \) we calculate the radiance as a function of \( \beta \). In the \( \beta \) space point \( O \) receives radiation between \( \beta_1 \) and \( \beta_2 \) (5.7. Fig.), where the hatched rectangle shows the area that illuminates into point \( O \). An infinitesimal surface \( dA \)
is drawn around point \( O \) in plane \( x-y \) (5.8. Fig.). First of all, we examine how an infinitesimal element of the lens aperture (\( \text{dx}\times\text{dy} \) in 5.7. Fig.) radiates into point \( O \). The direction vector \( \mathbf{OP} \) can be defined by angles \( \alpha \) and \( \beta \). As in the previous cases, \( \alpha \) is the plane angle between \( \mathbf{OP} \) and the normal vector of the exit pupil (vector \( z \)), and \( \beta \) is the plane angle between vector \( z \) and the projection of \( \mathbf{OP} \) onto plane \( y-z \). Provided that the distance between point \( O \) and the lens is much greater than the \( x \)-size of the lens aperture (\( d\gg D \)), we can accept that \( \alpha=\beta \). The elementary irradiance, emitted by the elementary surface \( \text{dx}\times\text{dy} \) of the lens into the observation point \( O \), can be calculated in the following way by using the power transfer equation [21.]

\[
\Delta \frac{dP}{dA} = R_s(x,\beta) \cdot \frac{\cos^4(\beta)}{d^4} \cdot dx \cdot dy
\]

5.9. Eq.

Expressing elementary source length \( dy \) with \( d\beta \) results in \( dy=\text{d}x\text{d}\beta/\cos^2(\beta) \). Substituting this formula into 5.9. Eq. gives:

\[
\Delta \frac{dP}{dA} = R_s(x,\beta) \cdot \frac{\cos^2(\beta)}{d} \cdot dx \cdot d\beta
\]

5.10. Eq.

The total irradiance can be calculated by integrating 5.10. Eq. on the rectangular area of the lens aperture that radiates into point \( O \) (5.11. Eq.).

\[
\frac{dP}{dA} = \int_{\beta_1}^{\beta_2} \int_{x_1}^{x_2} R_s(x,\beta) \cdot \frac{\cos^2(\beta)}{d} \cdot dx \cdot d\beta
\]

5.11. Eq.

5.8. Eq. defines parameters \( x_1 \) and \( x_2 \); parameters \( \beta_1 \) and \( \beta_2 \) are defined in 5.7. Fig.. The substitution of \( R_s(x,\beta) \) from 5.6. Eq. into 5.11. Eq. yields the following:

\[
\frac{dP}{dA} = \int_{\beta_1}^{\beta_2} \int_{x_1}^{x_2} R_s(x,\beta) \cdot \frac{\cos^4 \left( \frac{\sqrt{x^2 + f^2} \cdot \tan^2(\beta)}{f} \right)}{d \cdot \cos^2(\beta)} \cdot dx \cdot d\beta
\]

5.12. Eq.

The integral of 5.12. Eq. gives the irradiance at any point of the stripe. In general case this definite integral can be solved only numerically, however in particular cases the strong simplifications allow symbolical solutions. Anyway, the numerical evaluation of this definite integral is an effective tool for calculating the irradiance distribution of practical setups.
5.2.4 Lens parameters yielding maximal total light power

Stripe width is one of the factors limiting the accuracy of a stripe lighting sensor. The smaller stripe width is, the higher the achievable accuracy becomes. Therefore stripe lighting sensors require usually that the source should illuminate a stripe and the stripe width should be less than a given value inside a given distance range.

Our problem can be described in the following way: There is a rectangular emitter with width w. A stripe should be illuminated, and the stripe width should be less than S inside the distance range from 0 to d (5.3. Fig.). The question is what focal length (f) and aperture size (D) the cylindrical lens should have, in order to optimize the light power of the stripe. The cylindrical lens radiates divergent light, so the stripe width is equal to the lens aperture (D) at the lens and continuously increasing as the distance is increasing. The configuration is analyzed in which the stripe width reaches S at the maximal distance d.

Using 5.2. Eq. the numerical aperture of the lens can be calculated by the following formula:

\[
NA \approx \frac{D}{2 \cdot f} = \frac{S}{2 \cdot f} \cdot \frac{w \cdot d}{2 \cdot f^2}.
\]

Finding the extreme of NA supplies the focal length that couples the maximal light power into the required stripe geometry from a rectangular emitter of a given height (w). (5.13. Eq.).

\[
f = \frac{2 \cdot w \cdot d}{S}
\]

5.13. Eq.

Combining 5.13. Eq. and the formula of NA supplies the x aperture size (D) of the ‘optimal’ cylindrical lens:

\[
D = \frac{S}{2}
\]


5.3 SNR against sunshine for sources with different types of emitters

<table>
<thead>
<tr>
<th>Setup number</th>
<th>Source length (L)</th>
<th>Distance (d)</th>
<th>Stripe width (S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.4 m</td>
<td>1 m</td>
<td>0.01 m</td>
</tr>
<tr>
<td>2</td>
<td>0.4 m</td>
<td>1 m</td>
<td>0.05 m</td>
</tr>
<tr>
<td>3</td>
<td>0.4 m</td>
<td>2 m</td>
<td>0.01 m</td>
</tr>
<tr>
<td>4</td>
<td>0.4 m</td>
<td>2 m</td>
<td>0.05 m</td>
</tr>
</tbody>
</table>

Geometrical parameters of the four experimental sources

5.1. Table
Different applications need sources with different stripe width and illumination distance range. In this analysis preference is given to the requirements of mobile robot applications. The needed measurement distance range of a mobile robot depends on the moving speed of the robot. Stripe lighting measurement is not advantageous for measuring obstacles at large distances, since the irradiance decreases with increasing distance because of the divergence of the radiation. It is suitable for low and middle range measurements, in practice between 0 and 2-3 meters.

As all the possible variations of stripe width (S), distance range (d) and source length (L) cannot be evaluated, four experimental setups are selected (Table 1). All the parameters of the source are defined according to Fig 2 and Fig 3. In the next points these experimental sources are realized with three different types of emitters (LEDs, laser diodes, incandescent filaments) and the SNRs are calculated.

### 5.3.1 Definition of SNR

The SNR is defined as the ratio of the spectral irradiance (W/m²/nm) of the source and of the sun. The American Society for Testing and Materials (ASTM) defines two types of terrestrial spectral irradiance of the sun: The first is the direct normal spectral irradiance, the second is total spectral irradiance measured within a 2\(\pi\) steradian hemisphere [22.] (5.9. Fig.). For the SNR calculations the total solar spectral irradiance measured within a hemisphere is applied (5.15. Eq.).

\[
\text{SNR} = \frac{I_{\text{source}}}{I_{\text{sun total}}}
\]

5.15. Eq.

\(I_{\text{source}}\): Spectral irradiance [W/m²/nm] caused by the source

\(I_{\text{sun total}}\): Spectral irradiance [W/m²/nm] of the sun measured within a 2\(\pi\) steradian hemisphere
The irradiance of the source varies at different points of the stripe, thus the SNR also varies. For simplicity the SNRs are calculated in the center of the stripe (both in x and y directions) and at maximal distance \( z = d \) (Table 1). The SNR definition of 5.15. Eq. is valid only for perfect diffuse light scattering on the object and does not take the possibilities originating from reflection into account. Moreover the SNR of a complete sensor depends also on the construction of the imaging and detecting parts of the stripe sensor. For example if a spatial area that is bigger than the stripe width in direction x is imaged into one pixel of the image detector, then the achievable SNR degrades compared to 5.15. Eq.. This effect can be reduced by accurate sensor design.

5.9. Fig. shows that the spectral distribution of the solar irradiance has several minima, therefore using light sources emitting within the minima can result improved SNRs. The near IR range appears to be very promising, because of the minima and the wide variety of available high power and inexpensive sources. Our SNR calculations are based on the temporally continuous operation of the emitters, but real applications usually allow impulse driving, by which the achievable SNR can be increased. The sensitive spectral band of Si between 200nm and 1100nm is extremely important, since nowadays all low cost image detectors are made of Si.

### 5.3.2 SNR applying LED array

<table>
<thead>
<tr>
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<td>Siemens SFH4200</td>
<td>950</td>
<td>40</td>
<td>8.5</td>
<td>0.3x0.3</td>
<td>2360</td>
<td>3x3.4</td>
</tr>
<tr>
<td>CorkOpt CLED940-P-300X</td>
<td>940</td>
<td>50</td>
<td>1.75</td>
<td>0.3x0.3</td>
<td>389</td>
<td>3(?)</td>
</tr>
<tr>
<td>Optodiode OD-880C</td>
<td>880</td>
<td>80</td>
<td>4.45</td>
<td>0.35x0.35</td>
<td>454</td>
<td>3(?)</td>
</tr>
<tr>
<td>MARL 120081</td>
<td>660</td>
<td>40(?)</td>
<td>10.8</td>
<td>0.3x0.3</td>
<td>3000</td>
<td>3x2</td>
</tr>
<tr>
<td>Cree C525</td>
<td>525</td>
<td>35</td>
<td>1.43</td>
<td>0.3x0.3</td>
<td>420</td>
<td>3(?)</td>
</tr>
<tr>
<td>Cree C470</td>
<td>470</td>
<td>26</td>
<td>2.39</td>
<td>0.3x0.3</td>
<td>1000</td>
<td>3(?)</td>
</tr>
</tbody>
</table>

**Main parameters of LEDs used for the SNR calculations 5.2. Table**

The chip-type LEDs supplying the highest spectral radiance in different wavelengths were selected from the catalogs of different LED manufacturers. The most important parameters of these chip-type LEDs are summarized in Table 2 (see the web pages of the companies). In those cases, when a parameter was missing in the catalog, a standard value was taken. This is shown by a question mark in brackets (?) in the cells. The LED chips were supposed to be surface emitting types with Lambertian...
radiating characteristics [23.]. The first calculations are based on a rectangular emitter surface that is constructed of a continuous one dimensional array of LED chips. The realization of such LED chip array needs the availability of the single chips. The parameters of the cylindrical lenses (f and D) supplying the characteristics of Table 1 and the maximal light power were calculated according to 5.13. Eq. and 5.14. Eq. Only setup 2 leads to practically problematic solutions, where cylindrical lenses with NA over 1 should be applied. The other three setups require cylindrical lenses with maximal NAs around 0.5. The irradiance values caused by the source were numerically calculated using 5.12. Eq. that allowed the calculation of the SNR according to 5.15. Eq. 5.10. Fig. shows the calculated SNRs against the solar spectral irradiance for sources built of different LED chips. The best SNRs were achieved by the 950nm LED chip array. SNRs better than 2.9 were achieved at all four setups and setup 2 has an SNR of 35.7. The second best wavelength is 660nm, where all the SNRs are over one. These results show that a stripe illuminator built of an array of single LED chips and a cylindrical lens can supply SNRs greater than one for all the analyzed stripe geometries at temporarily continuous operation.

**SNR of experimental sources realized with an array of single LED chips against sunshine**

5.10. Fig.

LEDs are usually available in packaged and not in chip format. The construction of a custom LED chip array is economically reasonable only at high quantity production. Therefore the SNRs were also calculated for an emitter built of a linear array of packaged LEDs. For the Cree, Optodiode and CorkOpt LED chips, which are available in chip format, standard package sizes were assumed (Table 2). 5.11. Fig. shows the SNR values achieved by the different LEDs. The 950nm LED array supplies SNRs over 1.2 for three of the four stripe geometries. Further SNR improvements could be
achieved by the pulsed operation of the LEDs. This analysis leads to the conclusion that SNRs high enough for accurate measurements could be achieved even with stripe illuminators built of an array of packaged LEDs, however the selection of the appropriate LED type (wavelength, spectral radiance) is an essential criterion.

5.3.3 SNR applying laser diode array

Laser diodes (LD) are available in very wide spectral and power ranges. LDs with different internal structure have elliptical radiant intensity distributions of different types. The extremes of the far field intensity distribution for zero transversal order are exponential, Gaussian or cosine [24.]. I apply a two dimensional Gaussian distribution for the mathematical description of the LD’s angular radiant intensity characteristics:

\[
R(\theta_+, \theta_\parallel) = R \cdot 2 \left[ \frac{2\theta_+}{\theta_+^{\text{FWHM}}} \right]^2 \left[ \frac{2\theta_\parallel}{\theta_\parallel^{\text{FWHM}}} \right]^2
\]

5.16. Eq.

where

\( R(\theta_+, \theta_\parallel) \): Angular radiant intensity distribution [W/sr]

\( R \): Radiant intensity normal to chip surface

\( \theta_+ \): Radiation angle measured perpendicularly to the emitting stripe

\( \theta_\parallel \): Radiation angle measured parallel with the emitting stripe
$\theta_{I}^{\text{FWHM}}$: Angular full width of half maximum of the radiant intensity perpendicular to the emitting stripe

$\theta_{II}^{\text{FWHM}}$: Angular full width of half maximum of the radiant intensity parallel with the emitting stripe

The catalogs usually give only the parameters $\theta_{I}^{\text{FWHM}}$ and $\theta_{II}^{\text{FWHM}}$, but not the exact size of the emitting stripe. According to [25.] the dimensions of the emitting stripe can be calculated from the diffraction angle by 5.17. Eq., which can be regarded as an approximation of the diffracted intensity distribution of a rectangular aperture illuminated by a monochromatic plane wave [26.].

$$d \approx \frac{\lambda}{\theta_{I}^{\text{FWHM}}}$$

where

$\lambda$: wavelength

$d$: aperture size

The sizes of the emitting aperture of the LD’s were calculated from $\theta_{I}^{\text{FWHM}}$ and $\theta_{II}^{\text{FWHM}}$ using 5.17. Eq., LDs with 5mW power were selected in the visible and near IR spectral range. The technical parameters important for our purposes are summarized in Table 3 for the selected LDs (see the web pages of the companies). The determination of the spectral bandwidth of the laser diode array is a non-trivial issue. The bandwidth is constituted of the following three effects:

Emission bandwidth of the single LDs.
Emission wavelength variation of the different pieces of the same type LDs. Spectral sorting of the LDs can reduce this effect. The LDs of our experimental array is assumed to have an emission wavelength variation of 5nm achieved by spectral sorting, if necessary.

Emission wavelength shift of the LDs because of the ambient temperature changes. This can be reduced by temperature stabilization of the packages, although it is very expensive to accomplish for an LD array. For our experimental source a package temperature variation of 50K (no temperature stabilization) is supposed.

The full spectral bandwidth of the LD array can be calculated as a superposition of these three effects.

The LD’s are oriented in such a way that the junction is parallel with the y-axis, thus $\theta_1$ is measured in the xz plane and $\theta_{II}$ is measured in the yz plane (5.12. Fig.). The cylindrical lens configuration yielding maximal light power according to 5.13. Eq. and 5.14. Eq. cannot be applied, since a lens with extremely high numerical aperture would be needed because of the very small y size of the source (1-2µm). A lens aperture D approximately equal to the y size of the stripe can be used instead, as the radiation divergence behind the lens measured in the xz plane is very small. The acceptance angle of the lens is chosen to be equal with $\theta_{\text{FWHM}}$ (5.12. Fig.). Only the case is examined in which the emitter is constructed of an array of packaged LDs. The other case, an array of single LD chips can be omitted, since very good SNRs can be acquired even with low power packaged LD arrays. The SNRs were numerically calculated using 5.12. Eq., 5.15. Eq., 5.16. Eq. and 5.17. Eq. for all the LDs listed in Table 3 and they are shown in 5.13. Fig.. The 5mW LD array supplies SNRs better than one for all experimental stripe geometries at 405 and 950nm. The stripe geometries with 1cm stripe width supply SNRs better than 8 at these two wavelengths. The 785nm LD, used in CD readers, is very interesting.
for the light source construction, because of its low price. This LD gives SNRs better than 0.8 for all setups, and SNRs better than 4 for the stripes of 1cm width.

5.13. Fig.

The conclusion of this point is that the application of an array of low power (5mW) packaged laser diodes leads to SNRs near to or higher than one for different stripe geometries in several wavelengths. So this way of stripe illuminator construction yields both technologically and economically efficient solutions. LDs with higher than 5mW power are available at all wavelengths, consequently sources with higher SNRs can also be realized.

5.3.4 SNR applying incandescent source

The stripe illuminator can also be realized applying an incandescent source as an emitter. The radiance spectrum of a real incandescent source, which is assumed to be a Lambertian radiator, can be approximated using the characteristics of the gray body radiation [21.][27.]:

\[
R(\lambda, T) = e(\lambda, T) \cdot \frac{2 \cdot c^2 \cdot h}{h c} \lambda^5 \left(e^{\frac{h c}{\lambda k T}} - 1\right)
\]

5.18. Eq.

\(R(\lambda, T)\): Spectral radiance (measured in W/m²/sr/nm)
\(e(\lambda, T)\): Emissivity of the incandescent source
\( \lambda \): Wavelength  
\( T \): Temperature of the incandescent source  
\( c \): Velocity of light  
\( h \): Planck coefficient  
\( k \): Boltzmann coefficient  
The emissivity is not a constant, it is the function of the temperature and the wavelength. Different materials have different emissivity characteristics as well. Since the majority of the available incandescent sources are made of tungsten, this material is used in our analysis. The emissivity spectrum of Tungsten is given in [27.] at different temperatures. As Stefan’s low states, the total radiated energy of an incandescent source is proportional with the 4\(^{\text{th}}\) power of the temperature, thus incandescent sources of higher temperature should lead to improved SNRs.  
In our analysis a continuous rectangular tungsten surface of 1 mm height and 40cm length is applied. This form is merely theoretical, however a spiral of high fill factor wound around a rod with square cross-section approximates it quite well. Real linear tungsten emitters are either continuous rods or spirals, both with circular cross-section, so the SNRs calculated with rectangular emitter shape can be regarded only as approximations for these real forms. A real spiral is never continuous, so the losses originating from the fill factor should be also considered. Anyway the SNRs shown here can be regarded as theoretical maximums that probably cannot be overcome with real tungsten sources.  
Four experimental sources supplying the parameters of Table 2 are realized with the rectangular emitter applying the lens parameters of maximal light power (5.13. Eq., 5.14. Eq.). The SNR spectra are calculated for the different incandescent temperatures of 2500K and 3000K and the results are shown in 5.14. Fig. and 5.15. Fig.
SNR of experimental sources realized with a rectangular tungsten emitter of 1 mm height at 2500K

5.14. Fig.

SNR for sources with 2500K tungsten emitter

SNR of experimental sources realized with a rectangular tungsten emitter of 1 mm height at 3000K

5.15. Fig.
The application of incandescent sources would supply technological advantages over LEDs or LDs, only if a wide spectral band of the emission could be applied. Firstly, this would lead to higher power stripe illuminators than the ones realized with LEDs or LDs. The 2500K lamp has wide spectral bands with SNRs greater than one only over 1100nm (5.14. Fig.), which would require the application of expensive image detectors. As 5.15. Fig. shows the 3000K lamp supplies SNRs higher than one over 540nm for setup 2, so it is a very high total power stripe illuminator. For setup 1 the SNR exceeds one over 880nm, and for setup 4 over 800nm. These sources can be also considerable for certain applications. For setup 3 the SNR exceeds one only over 1100nm, thus it is a rather ineffective source. Tungsten lamps can be operated up to 3400K yielding higher SNRs for the cost of the lifetime, which significantly degrades at these high temperatures.

5.4 Conclusions

In this point the results of the previous examinations are summarized. The radiometric analysis of the stripe source, based on a cylindrical lens and a spatially homogenous rectangular emitter placed at the focal line, was accomplished applying the model of an ideal cylindrical lens. The result of the analysis is an integral that can be applied for any real source configuration. The focal length and aperture height of the cylindrical lens were determined that couples the maximal light power from an emitter of specific size into a stripe with a specific geometry. Based on these general results experimental stripe illuminators were examined by applying LEDs, laser diodes and incandescent sources as emitter. The SNRs of the experimental sources against direct sunshine were calculated and different stripe sources were proposed for outdoor applications.

Only the 950nm LED supplies good SNRs within the group of the packaged LED solutions, the second best is the 630nm LED. The application of these LEDs yields very economical outdoor stripe illuminators. Higher SNRs can be gained by applying custom arrays of LED chips for increased construction costs. The first disadvantage of LED solutions is that the narrow spectral band emission of LEDs could lead to significant signal level variations at objects of different colors. The inaccurate manufacturing of the LEDs could cause construction difficulties, which could be solved by sorting of the LEDs (see point 4.3.3).

The 5mW laser diodes at all the examined wavelengths supply good SNRs. The application of low cost 670nm or 785nm LDs yields economically also effective solutions. LDs with higher than 5mW power are advantageous for applications requiring high SNRs. The main disadvantages of a stripe illuminator built of LDs are the emitted narrow spectral band and the potential eye injury hazard.

Only the application of high temperature (over 3000K) tungsten incandescent sources supply good SNRs, but the width of the usable spectral band depends strongly on the geometry of the illuminated stripe. The main technological advantage of this source type is that a wide spectral band of the emission (e.g. over 540nm) can be used for certain stripe geometries. This could lead to stripe illuminators of high total power and reduce the signal variations originating from object color.
Another positive feature of these stripe sources is the low price of tungsten filaments. The main disadvantage is the low power efficiency, since only a small portion of the total illumination can be coupled into the stripe because of the spectral and directional losses. The low accuracy of tungsten filament mounting could cause construction problems.

The application of gas discharge lamps is not discussed in this chapter because of their very wide variety and the lack of generalised physical interpretation of their radiation characteristics. Probably they can be also applied in outdoor applications, since some of them (e.g. high pressure Xe lamp) supply higher spectral brightness than conventional incandescent sources.

The main goal of this analysis was to decide which stripe illuminator configurations were suitable for outdoor applications. The analysis concentrated on the calculation of the achievable SNR against solar irradiation, which is a basic difficulty in outdoor applications. The SNR calculations presented here give good approximations for what can be achieved by the different configurations. Further features of the source, as total light power, spectral broadness, eye safety and construction difficulties should be taken into account in order to find the most appropriate solution for the specific needs of an application. These problems could be only very briefly discussed here; they should be analyzed much thoroughly during the design and the construction of a specific version.
6 Ternary phase-amplitude modulation with transmission type twisted nematic liquid crystal displays for Fourier holographic data storage

6.1 Introduction

Holographic data storage is an emerging technique within the field of optical information storage. A holographic system converts several bytes of information into a data image and stores it in a hologram. The hologram is created as the interference of a reference beam and an object beam, where the object beam is amplitude modulated according to the data image (see 6.1. Fig.). The amplitude modulation is accomplished by spatial light modulators such as liquid crystal displays or micro electro mechanical mirror arrays. In order to achieve optimal data density the hologram at the back focal plane of the object lens (Fourier plane) is stored. An important problem of Fourier holography is that the zero spatial frequency component of the Fourier transform can be much higher than other frequency components, thus storage material with very high dynamic range is required. A well known technique to suppress the zero frequency component and to homogenize the intensity variations of the Fourier plane is the random phase modulation of the light passing through the different pixels of the object [28.]. This is usually accomplished by external pixelated random phase modulating masks. These masks have the same pixel number and geometry as the applied spatial light-amplitude modulator and the two devices are imaged onto each other and pixel by pixel position matched (see 6.1. Fig.). Each pixel of the mask gives a random phase modulation thus the zero frequency component is effectively destructed. The pixel matched positioning of the phase mask and SLM requires an aberration free relay lens and a complicated alignment in six axes leading to very complex and expensive optical systems.
Another possibility is to integrate an external phase mask into the SLM without a relay lens, however this solution needs to place the phase mask very close to the light modulator plane (0.1-0.2 mm) to be within the focal depth determined by the diffraction on the pixels (see 6.2. Fig.). The usual cover glass width (~1mm) of commercially available SLMs does not enable to directly apply this solution, so the modification of the SLMs or custom SLMs are required. The positioning and fixing of the mask is also a difficult operation because of the needed micron or submicron accuracy. According to our knowledge only the company Displaytech supplies a Ferroelectric device in which a micro lens array with random phase modulation is integrated into the SLM. Another drawback of the application of static phase modulating masks is that the zero frequency component is not destructed for certain data bit patterns in which the transmitting SLM pixels well correlate with the phase modulating pixels of similar phase modulations. Therefore these data bit patterns should be avoided by coding techniques, which leads to data capacity losses. Because of the previously described difficulties it would be rather advantageous to realize the required simultaneous amplitude and phase modulation within the single SLM pixel. In this chapter I investigate the possibilities to realise such a simultaneous amplitude and phase modulation of SLM-s applying transmission type twisted nematic liquid crystal displays.

The theoretical backgrounds of Fourier plane intensity smoothing by pixelated phase modulation is summarized in [28.]. Generally the amplitude modulation can be either multi level or binary enabling gray scale or binary data images. In an ideal phase modulating pixel array that optimally destructs the zero order each pixel has a random phase modulation between 0 and 2π. Another method is the application of pseudo-random phase modulating arrays in which the maximal phase step between the adjacent pixels is restricted. This solution yields increased data storage density compared to random phase masks. Real phase masks are realized with a certain number of phase steps between 0 and 2π. The easiest phase mask construction is a binary phase mask with the two phase steps of 0 and π; a phase mask with four steps yields the phase modulation of 0, 0.5π, π and 1.5π. Increasing the number of phase steps approximates better the analog phase modulation, consequently enhances Fourier spectrum smoothing and reduces image noise.
Hence the ideal SLM pixel should supply analogous modulation of both amplitude and phase, however such a modulation requires two LC cells (e.g. one working in amplitude only, the other in phase only mode). In this chapter operating modes of transmission type twisted nematic (TN) LCDs are investigated that supply two bright states with equal amplitudes and $\pi$ phase difference and a dark state with much lower amplitude. Such an operating mode with three states is able to display binary data images with a suppressed zero frequency component. This modulation scheme is already known as Ternary Phase-Amplitude (TPA) Filter ($+1,-1,0$) in the literature of correlation filters [29.]. The realisation of more complicated modulation modes such as e.g. Penta Phase-Amplitude ($1, e^{j\pi/2}, -1, e^{-j\pi/2}, -1,0$) or Quaternary Phase-Amplitude ($+1,-1,\epsilon,-\epsilon$) is probably also possible, however in this chapter I concentrate on the TPA modulation.

### 6.2 Derivation of the Jones matrix describing both the polarisation and phase modulation of a transmission type twisted nematic LC cell

There are three different phases of liquid crystal material such as smectic, nematic and cholesteric. The most important of them for commercial LC displays is the nematic phase so I will discuss only this phase. Liquid crystal is generally composed of rod shaped molecules. In the nematic phase the rod-like molecules are oriented “parallel” with a specific direction called “LC director” (see 6.3. Fig./a). Beside this orientational order there is no other order coordinating the placement of the molecules, thus the molecules are randomly positioned. A very special, artificially constructed version of it is the twisted nematic phase which is composed of “planar layers” of nematic phase and the LC director is continuously twisting in the consecutive layers (see 6.3. Fig./b). Such a twisted nematic phase is usually sandwiched between two parallel glass plates and the glass surfaces touching with the liquid crystal are rubbed in different directions. The LC directors are forced to be parallel with the
rubbing directions at each glass surfaces and homogenously twisting in between. The LC material has uniaxial birefringence and the axis is parallel with the director.

If a twisted nematic LC medium is illuminated with polarised light, which is usually linear but can be elliptical as well, both the polarisation and phase of light are modulated by the LC medium. The literature considering the twisted nematic medium focuses usually only on the polarisation modulation since this is the most important for creating displays for intensity modulation of light. Since the phase of light has no significance for the eye so the phase modulation of the LC medium is neglected [31.][32.]. This problem has already induced some misunderstandings, e.g. in [30.] the authors applied the Jones matrix describing only the polarisation modulation for calculating both the polarisation and phase modulation of a TN LC medium. The correct formulas describing both polarisation and phase can be obtained simply by following the steps of the derivation of the Jones matrix of polarisation modulation with taking also the phase into account.

Let’s consider a twisted nematic medium where the LC directors are perpendicular to z and twisted around axis z (see 6.4. Fig.). As it has been already mentioned a nematic liquid crystal material has uniaxial anisotropy and the symmetry axis is parallel with LC director. Let’s denote the extraordinary and ordinary refraction indices with $n_e$ and $n_o$ respectively as 6.4. Fig. shows. $n_e$ denotes the refraction index for the linear polarisation parallel with LC director and $n_o$ perpendicular to it. The ellipse defined by $n_e$ and $n_o$ twists simultaneously with the homogenously twisting LC director. Let’s denote the thickness of the LC cell in direction z by $L$ and the total twist angle of the LC molecules in the whole LC medium by $\alpha$. The modulation of the LC cell for a plane wave of general elliptical polarisation and wavelength $\lambda$ propagating parallel with axis z will be examined. A new parameter $\Gamma$ denoting the phase retardation of the nematic LC medium in untwisted state can be defined as follows:

$$\Gamma = \frac{2 \cdot \pi}{\lambda} (n_e - n_o) \cdot L$$

6.1. Eq.

Let’s divide the whole LC medium into N equally thick layers in direction z. The plates can be regarded as birefringent layers of $\Delta \Gamma = \Gamma / N$ phase retardation with homogenously twisting
extraordinary axes. The Jones matrix of a single layer relative to co-ordinate axes defined by the
extraordinary and ordinary polarisation directions can be written on the following way:

\[
W_0 = \begin{bmatrix}
-\frac{j 2\pi}{N} \frac{n_e L}{2}
& 0 \\
0
& \frac{j 2\pi}{N} \frac{n_o L}{2}
\end{bmatrix}
= e^{-\frac{j 2\pi}{N} \frac{n_e L}{2}} \cdot
\begin{bmatrix}
-\frac{j 2\pi}{N} \frac{n_o L}{2}
& 0 \\
0
& \frac{j 2\pi}{N} \frac{n_e L}{2}
\end{bmatrix}
= e^{-\frac{j 2\pi}{N} \frac{n_o L}{2}} \cdot
\begin{bmatrix}
0
& e^{\frac{j \pi}{2N}} \\
e^{\frac{j \pi}{2N}}
& 0
\end{bmatrix}
\]

6.2. Eq.

Each layer has a Jones matrix given by 6.2. Eq. and the extraordinary axes subtend angles \( \alpha / N, 2 \alpha / N, \\
3 \alpha / N, \ldots, \alpha \) with axis x. The Jones matrix of a such a layer structure can be described with 6.3. Eq.

\[
\mathbf{W} = \prod_{m=1}^{N} \mathbf{R}(-m \cdot \frac{\alpha}{N}) \cdot \mathbf{W}_0 \cdot \mathbf{R}(m \cdot \frac{\alpha}{N})
\]

6.3. Eq.

where \( \mathbf{R}(\alpha) \) denotes the transformation matrix into another co-ordinate system rotated with angle \( \alpha \)
and \( \mathbf{R}(\alpha) \) can be described on the following way:

\[
\mathbf{R}(\alpha) = \begin{bmatrix}
\cos(\alpha) & \sin(\alpha) \\
-\sin(\alpha) & \cos(\alpha)
\end{bmatrix}
\]

6.4. Eq.

The product of the consecutive two elements of the matrix product of 6.3. Eq. is constant for each
\( m=1,2,\ldots,N \) as it is shown below:

\[
\mathbf{R}(m \cdot \frac{\alpha}{N}) \cdot \mathbf{R}(-(m-1) \cdot \frac{\alpha}{N}) = \mathbf{R}(\frac{\alpha}{N})
\]

Using this relation 6.3. Eq. can be rewritten into the following form:

\[
\mathbf{W} = \mathbf{R}(-\alpha) \cdot \left( \mathbf{W}_0 \cdot \mathbf{R}(\frac{\alpha}{N}) \right)^N
\]

6.5. Eq.

By substituting 6.2. Eq. and 6.4. Eq. into 6.5. Eq. we obtain:

\[
\mathbf{W} = \mathbf{R}(-\alpha) \cdot e^{-\frac{j 2\pi n_o L}{N}} \cdot
\begin{bmatrix}
\cos(\frac{\alpha}{N}) \cdot e^{-\frac{j \pi}{2N}} & \sin(\frac{\alpha}{N}) \cdot e^{-\frac{j \pi}{2N}} \\
-\sin(\frac{\alpha}{N}) \cdot e^{\frac{j \pi}{2N}} & \cos(\frac{\alpha}{N}) \cdot e^{\frac{j \pi}{2N}}
\end{bmatrix}^N
\]

6.6. Eq.
It can be shown (see Appendix II) that when $N$ tends to infinity the limit of the above expressions is:

$$
W = R(-\alpha) \cdot e^{\frac{2 \pi}{X} \cdot \text{sgn}(N)} \cdot \left[ \begin{array}{c}
\cos(X) - j \cdot \frac{\Gamma \cdot \sin(X)}{2X} \\
- \alpha \cdot \frac{\sin(X)}{X} \\
\alpha \cdot \frac{\sin(X)}{X} \\
\end{array} \right]
$$

where parameter $X$ denotes:

$$
X = \sqrt{\alpha^2 + \frac{\Gamma^2}{4}}
$$

The Jones matrix defined by 6.7. Eq. describes the polarisation and absolute phase modulation of a twisted nematic LC cell.

The response of nematic LC medium to external electrical field

a) Nematic LC phase in electrical field under the minimal threshold
b) Nematic LC phase in electrical field between the minimal and maximal thresholds
c) Nematic LC phase in electrical field over the maximal threshold

In the remaining part of this point the response of a TN LC layer to electric field will be briefly discussed. Most LC materials have positive uniaxial birefringence, namely the extraordinary refractive index is bigger than the ordinary. The symmetry axis of the index ellipsoid is parallel with the LC director, thus the maximal extraordinary refractive index can be measured for polarisation parallel with the LC director as well. Because of Maxwell’s relation ($\varepsilon = n^2$) the dielectric constant takes its maximum also in the direction parallel with the LC director, thus the electrical dipole polarizability of the molecules due to the external electrical field is also maximal in this direction. By applying
The induced dipole polarisation in the LC molecules will not be parallel with the electrical field because of the anisotropy, thus an electrically induced torque rotates the molecules to make the dipole polarisation vector parallel with the electrical field. In reality it means that the LC director rotates to become parallel with the electrical field, since $\varepsilon$ is maximal parallel with the LC director. The internal elastic forces of the LC material counteract this rotation of the molecules and in equilibrium the electrical and elastic torque are equal. This process is shown very simply in 6.5. Fig. In 6.5. Fig./a is shown that if the electrical field is below a threshold value, denoted by $E_{\text{min}}$, the LC molecules do not rotate and the LC material is in its original state. In 6.5. Fig./c is shown that if the electrical field is over another threshold value, denoted by $E_{\text{max}}$, then all the rod-like LC molecules are parallel with the electrical field. If an electrical field between $E_{\text{min}}$ and $E_{\text{max}}$ is applied, as it is shown in 6.5. Fig./b, the LC molecules are in an intermediate state, but henceforward they form a nematic phase.

Let’s consider a plane wave propagating parallel with the electrical field. Since the symmetry axis of the uniaxial index ellipsoid is parallel with the LC director (and with the rod-like molecules) the extraordinary refractive index will be changed by the applied electrical field and the ordinary refractive index remains constant for this light wave. In the state of zero field the extraordinary refractive index takes its maximal value. At an electrical field over the maximal threshold the light is propagating parallel with the symmetry axis of the index ellipsoid, thus the LC material has no birefringence in this state, consequently both refractive indices are equal to no. By applying an electrical field between the minimal and maximal thresholds the extraordinary refractive index can be changed between ne and no.

The above considerations were made for nematic LC material, however the same is true for a twisted nematic one. In the Jones matrix obtained for the TN LC cell, specified by 6.6. Eq. and 6.7. Eq., $n_e$ can be modulated by switching electric field onto the cell. 6.1. Eq. shows that the parameter $\Gamma$ is dependent on $n_e$. At zero field $\Gamma$ takes its maximal value and at a field over the maximal threshold $\Gamma$ is equal to zero since $n_e$ is equal to no. The other parameter depending on $n_e$ is the exponential phase modulation factor in 6.7. Eq. If a relative phase modulation is acceptable instead of the absolute phase, 6.7. Eq. can be simply rewritten into the following form:

$$W = R(-\alpha) \cdot e^{-j \frac{\pi}{2}} \cdot e^{-j \frac{2\pi}{\lambda} n_o L} \cdot \begin{bmatrix} \cos(X) - j \cdot \frac{\Gamma}{2} \cdot \frac{\sin(X)}{X} \cdot \frac{\sin(X)}{X} \\ -\alpha \cdot \frac{\sin(X)}{X} \cdot \cos(X) + j \cdot \frac{\Gamma}{2} \cdot \frac{\sin(X)}{X} \end{bmatrix}$$

6.9. Eq.

The advantage of the above formula is that the only parameter changed by electrical field is $\Gamma$, but parameter $X$ depends also on $\Gamma$. Since the LC material has uniaxial birefringence, the ordinary refractive index (no) does not depend on the orientation of the molecules, so it is not modulated by the electrical field. Therefore the constant exponential factor $e^{-j \frac{2\pi}{\lambda} n_o L}$ is neglected in the later calculations. If only the polarisation modulation is considered then the exponential phase modulating
element $e^{-j\Gamma/2}$ can be neglected as well, as it is usually done in the basic literature [31.][32.]. However if the phase modulation is also examined then the exponential factor of $e^{-j\Gamma/2}$ should be also taken into account.

The above model was established for ideal uniformly twisting birefringent media. According to [36.] there is a very thin LC layer adjacent to the glass plate (see 6.1. Fig.) in which the LC molecules are not rotated by the applied electrical field. The thickness of this layer decreases by increasing electrical field. [36.] presents a very simple model to describe this effect, however the measurement of new parameters are needed at different pixel voltages. As the later points show the simpler model for ideal TN LC material was also relevant to realise the required operating modes.

Let’s consider the case when $\alpha \ll \Gamma$. In this case $X=\Gamma/2$, thus the Jones matrix specified by 6.9. Eq.

\[
W = R(-\alpha) \cdot e^{-j\Gamma/2}.
\]

If this Jones matrix is multiplied with a linear polarisation defined either by $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ or by $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$, then the linear polarisation is rotated with angle $\alpha$. This effect is called adiabatic following, namely if an LC cell with $\alpha \ll \Gamma$ is illuminated with a linear polarisation either parallel with or perpendicular to the local LC director, then the LC cell rotates the linear polarisation with the internal twist angle ($\alpha$).

Many of the commercially available transmission type TN LCDs have $\pm 90^\circ$ internal twist angle and it is sandwiched between two orthogonal linear polarisers with polarizing axes parallel with or perpendicular to the local LC directors at the two surfaces of the LCD. So at maximal $\Gamma$ (at zero pixel voltage) the incident linear polarization is rotated by $\pm 90^\circ$ and transmitted by the second linear polariser, if the LCD realizes adiabatic following. At maximal voltage the LCD pixels are not birefringent thus the incident polarization is not rotated by the LC material so it is blocked by the second linear polariser. By varying the pixel voltages from zero to maximal voltage the intensity transmission of the LCD pixels can be modulated.
6.3 Generation and detection of arbitrary elliptical polarisations

In this point I will show that by a sequence of a revolving polariser and a revolving quarter wave plate arbitrary elliptically polarised illumination can be generated for the LCD (see 6.6. Fig.). Similarly by the application of a sequence of a revolving quarter wave plate and a revolving linear analyser the light leaving the LCD can be separated to an arbitrary pair of orthogonal elliptical polarisations, one of them is fully transmitted, the other is blocked by the analyser (see 6.6. Fig.).

Let’s denote the rotation angle from axis x to the polarising axis of the polariser with \( p \), and the angle from axis x to the slow axis of the quarter wave plate with \( q_1 \) (see 6.6. Fig.). The elliptical polarisation generated by these two elements can be formulated as (\( R(x) \) denotes the rotation matrix defined by 6.4. Eq.):

\[
\begin{bmatrix}
E_x \\
E_y \\
\end{bmatrix} = R(-q_1) \cdot \begin{bmatrix} 1 & 0 \\ 0 & j \end{bmatrix} \cdot R(q_1) \cdot R(-p) \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix}
\]

6.11. Eq.

Applying a new parameter \( s = q_1 - p \), which denotes the angle from the polarising axis of the polariser to the slow axis of the quarter wave plate, the above equation can be rewritten in the following form:

\[
\begin{bmatrix}
E_x \\
E_y \\
\end{bmatrix} = R(-q_1) \cdot \begin{bmatrix} 1 & 0 \\ 0 & j \end{bmatrix} \cdot R(s) \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix}
\]

After evaluating the matrix products we receive the following formula:
\[
\begin{bmatrix}
E_x \\
E_y
\end{bmatrix} = R(-q1) \cdot \begin{bmatrix}
\cos(s) \\
-j \cdot \sin(s)
\end{bmatrix}
\]


The two elements of the vector \[
\begin{bmatrix}
\cos(s) \\
-j \cdot \sin(s)
\end{bmatrix}
\]
specify the complex amplitudes of an elliptically polarised wave relative to the axes of the ellipse, since there is \( \pi \) phase difference between them. The ratio of the amplitudes is \( \cos(s)/(\sin(s)) = \cotg(s) \). By varying \( s \) from \( -\pi/2 \) to \( \pi/2 \), \( \cotg(s) \) is varied from \( -\infty \) to \( \infty \), consequently any ratio between the two axes of the ellipse can be established. In 6.12. Eq. this ellipse is multiplied with the rotation matrix \( R(-q1) \), thus by changing \( q1 \) between 0 and \( \pi \) the elliptical polarisation can be rotated to any direction. These considerations prove that by the sequence of a revolving linear polariser and quarter wave plate any elliptical polarisation can be generated.

The light wave incident on the analyser can be calculated from the light leaving the LCD on the following way (\( q2 \) is the angle subtended by the slow axis of the 2\textsuperscript{nd} quarter wave plate and axis \( x \), see 6.6. Fig.):

\[
\begin{bmatrix}
E_{anx} \\
E_{any}
\end{bmatrix} = R(-q2) \cdot \begin{bmatrix}
1 & 0 \\
0 & j
\end{bmatrix} \cdot R(q2) \cdot \begin{bmatrix}
E_{lcdx} \\
E_{lcdn}
\end{bmatrix}
\]

6.13. Eq.

In general case the light wave leaving the LCD, denoted by \[
\begin{bmatrix}
E_{lcdx} \\
E_{lcdn}
\end{bmatrix},
\]
is elliptically polarised. This elliptically polarised light can be described as a sum of two orthogonal elliptical polarisations [34.].

The two orthogonal elliptical polarisations, relative to co-ordinate axes parallel with their major and minor axes, can be written in the following forms:

\[
\begin{bmatrix}
E_1 \\
E_2 \cdot j
\end{bmatrix}
\text{ and } \begin{bmatrix}
-E_2 \cdot j \\
E_1
\end{bmatrix}
\]

where \( E_1 \) and \( E_2 \) are real numbers\(^\circ\). As the above polarisation vectors show the major and minor axes of the two orthogonal elliptically polarised components are exchanged and they are rotating in the opposite direction. We would like to separate these two orthogonal elliptical components, namely one of them should be detected without polarisation loss and the other one should be completely blocked. To separate them the quarter wave plate should be rotated to that angular setting, specified by \( q2 \), in which the slow and fast axes are parallel with major and minor axes of the orthogonal elliptical component to be detected. This can be done in two angular settings of the quarter wave plate described by 6.14. Eq. and 6.15. Eq.

\(^\circ\)Two orthogonal complex vectors can be written in the following form: \([A, B]\) and \([B^*, -A^*]\). \( A^* \) denotes the complex conjugate of \( A \). The scalar product of the two vectors is \( AB \cdot BA = 0 \).
\[
\begin{bmatrix}
E_1 \\
-E_2
\end{bmatrix} =
\begin{bmatrix}
1 & 0 \\
0 & j
\end{bmatrix}
\begin{bmatrix}
E_1 \\
E_2 \cdot j
\end{bmatrix}
\]
\[
\begin{bmatrix}
-E_2 \cdot j \\
-E_1 \cdot j
\end{bmatrix} =
\begin{bmatrix}
1 & 0 \\
0 & j
\end{bmatrix}
\begin{bmatrix}
-E_2 \cdot j \\
-E_1
\end{bmatrix}
\]


\[
\begin{bmatrix}
E_2 \\
-E_1
\end{bmatrix} =
\begin{bmatrix}
j & 0 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
E_1 \\
E_2 \cdot j
\end{bmatrix}
\]

6.15. Eq.

In both settings the two orthogonal elliptical components are converted into two orthogonal linear components by the quarter wave plate as 6.14. Eq. and 6.15. Eq. show, since $E_1$ and $E_2$ represent real numbers. Consequently both settings of the quarter wave plate are able to separate the two orthogonal elliptical polarisations into two orthogonal linear polarisations, which can be perfectly detected and blocked by the analyser.

Next I will find in which angular areas the quarter wave plate and analyser should be rotated to detect all possible elliptical polarisations. If the quarter wave plate is rotated in an angular range of $0-\pi/2$ the fast and small axes become parallel with the axes of the ellipse to be detected in one angular setting, where the orthogonal elliptical components are separated into orthogonal linear components. By rotating the analyser in the angular range of $0-\pi$ any of them can be detected. The same is true for the generation of elliptical polarisations: by rotating the polariser in the angular range of $0-\pi$ and the quarter wave plate between $0$ and $\pi/2$ any elliptically polarised light can be generated.
6.4 TPA modulation setup of Jang and Shin

Jang and Shin published a configuration using a monochrome Kopin LCD with 320x240 pixels [30.] to realise TPA modulation. The Kopin LCD with -90° twist angle was assumed to realise adiabatic following [31.][32.]. They applied a single polariser on the incident side and a quarter wave plate plus an analyser on the exit side (6.7. Fig.). The transmitting axis of the polariser was parallel with the local LC director of the LCD. The slow axis of the quarter wave plate subtended 45° with the extraordinary axis of the LCD on the incident side. In their setup only the analyser angle was changed in order to realise TPA modulation. Their method to find the TPA states can be summarised on the following way. An ideal TN LC material (for which adiabatic following is true) with 90° twist angle gives two orthogonal linear polarisations with the same amplitude for minimal and maximal driving voltages for this type of linearly polarised illumination. These opposite linear polarisations become two oppositely rotating circular polarisations after the quarter wave plate. By changing the direction of the analyser such a direction can be found, in which the projections of the two oppositely rotating circular polarisations totally destruct each other. In this analyser setting the responses of the LC pixels to minimal and maximal voltages have the same amplitude modulation with π phase difference, thus two states of TPA modulation can be realised (+1,-1). It should be pointed out that half of the light power is absorbed by the analyser, since both waves are circularly polarised. Finding that intermediate pixel voltage, for which the LC cell gives the minimal amplitude, the third TPA state may be found (0), however Jang and Shin presented neither theoretical nor experimental guarantee for the existence of a dark state with small enough amplitude. Jang and Shin found experimentally such an analyser setting in which the two bright states totally destructed each other, but nothing is written about the
amplitude of the dark state at this setting. At another analyser alignment the system yielded a dark state with high contrast but there was only 0.73π phase difference between the two bright states here. The main drawbacks of this method of realising TPA modulation are: It guarantees only the existence of the two required bright states (+1,-1), but neither theoretical nor experimental proof for the existence of a dark state with high contrast was presented. Furthermore half of the light power originating from the two bright states (+1,-1) is necessarily lost on the analyser. These drawbacks can be originated in the inherent limitations of the method. By this optical setup and by varying only the direction of the analyser axis just a very small part of the possible incident and exit elliptical polarisations are examined, because the incident and exit elliptical polarisations make a four dimensional space as it was shown in point 6.3. By exploiting the whole 4 dimensional parameter space of input and output polarisations TPA modulation can be more effectively realised, as it is shown later in this chapter.

As I have already mentioned Jang and Shin made an error in the theoretical modelling, namely they applied the Jones matrix (6.9. Eq.) excluding the exponential phase modulation factor to calculate both the phase and polarisation modulation of the LCD.

6.5 Ternary Phase-Amplitude modulation applying transmission type twisted nematic LC cell

From this point I concentrate on the problem of finding the incident and exit elliptical polarisations optimal for TPA modulation by numerically evaluating the modulation of the LC cell using the Jones matrix model presented. The optical configuration applied for the simulations and measurements is shown in 6.6. Fig. The extraordinary axis of the LCD at the incident side is chosen to be parallel with axis x. For the simulations the twist angle of the LC molecules is accepted to be –90°, since very similar rotations are applied in most of the commercial TN LCDs. According to the Jones matrix (6.9. Eq.) a TN cell with higher \( \Gamma \) has an extended “modulation space” compared to a TN cell with smaller \( \Gamma \). This means that a cell with higher \( \Gamma \) can realize all the modulation capabilities of a cell with lower \( \Gamma \) and much other. Therefore for LC cells with different \( \Gamma \) values different optimal input and output polarisations are expected.

6.5.1 Ternary phase-amplitude modulation with \( \Gamma=5.2 \)

In the first computer search a phase retardation of \( \Gamma =5.2 \) was chosen, since according to our measurements using the method published in [35.], the Sony LCX017DLT display supplied this \( \Gamma \) value at 532nm wavelength. The modulation characteristics of the LC cell were evaluated at points of the four dimensional space (angular settings of polariser, analyser and 2 wave plates in setup of 6.6. Fig. with 5° resolution along each axes. In each configuration the amplitude of illumination after the polariser was one (see 6.6. Fig.). Those configurations were accepted that supplied the two bright states with amplitude ratio of 0.95-1.05 and phase difference of 0.95π-1.05π; and also supplied a dark state (0) with intensity contrast better than 15 compared to the bright states (+1,-1). The
computer search resulted a huge amount of configurations matching the criteria. Another important parameter is the light loss defined as the fraction of light originating from the bright (+1,-1) pixels blocked by the analyser. The smaller the light loss, the smaller power laser should be used in the writer device, thus economically it is a very important parameter. All the configurations, supplying the required three modulation states with maximum light amplitude (there were several such configurations), supplied amplitudes close to 0.5 in the bright states. The calculated amplitude and phase modulation characteristics of one such configuration, $p=160^\circ$, $a=90^\circ$, $q_1=130^\circ$, $q_2=30^\circ$ (parameters defined in 6.6. Fig.), are shown in 6.8. Fig./a. As the curve shows, it supplies theoretically two bright states with $\pi$ phase difference at amplitudes of 0.48 and 0.5, and the dark state has an amplitude of 0.12.
a) Calculated amplitude and phase modulation characteristics of TN LC cell with $\Gamma = 5.2$ in the configuration: $p = 160$, $a = 90$, $q_1 = 130$, $q_2 = 30$

b) Measured amplitude and phase modulation characteristics of Sony LCX017DLT TN LCD with 532nm laser in the configuration: $p = 160$, $a = 90$, $q_1 = 130$, $q_2 = 30$
Test measurements were done to test this configuration, by applying a 100mW 532nm laser, Polaroid HN38S linear polarisers for polariser and analyzer, a true zero order crystal quarter wave plate at the incident side and a quarter wave retarder foil (nominal retardance: 140nm, type IP 140WR01 from International Polariser) at the exit side. The retardance of the IP 140WR01 foil was exclusively measured at 532nm wavelength resulting 129nm retardance. The Sony LCX017DLT display was driven by a custom electronics. The measured amplitude and phase modulation characteristics as functions of the pixel voltage are shown in 6.8. Fig./b. The relation between pixel voltage and $\Gamma$ of the cell was not determined since it has no particular interest to this examination. In the measurements there were two bright states at amplitudes of 0.44 and 0.42 with $\pi$ phase difference and the amplitude of the dark state was 0.11. Thus the measurements coincide with the numerical calculations quite well. The deviation between measurements and theory are caused by several reasons: a real TN LC cell is not a uniformly twisting medium, there are deviations mainly close to the sides [36.]; there can be a slight difference between the accepted twist angle of 90° and the real twist angle; for the orientation of the extraordinary axis of the LC material the catalogue value was accepted, however there can be a slight difference; the retarder foil at the exit side of the LCD was not an ideal quarter wave plate. The phase modulation of the LCD was measured by the method published in [37].

6.5.2 Ternary phase-amplitude modulation with $\Gamma=6$

In the next computer search $\Gamma=6$ was chosen, because this $\Gamma$ value was measured at 488nm wavelength using the LCX017DLT display. This computer search for TPA modulation resulted a huge number of configurations, the ones with the highest amplitude supplied amplitudes between 0.6 and 0.7 in the bright states. The numerically calculated and measured amplitude and phase modulation characteristics of one of these configurations, $p=110$ $a=100$ $q1=35$ $q2=45$, are shown in 6.9. Fig./a-b respectively. As 6.9. Fig./a shows, theoretically this configuration supplies two bright states with amplitudes of 0.6 and phase difference of $\pi$; the amplitude of the zero state is 0.15, thus the intensity contrast is 16. In the measured amplitude and phase modulation curves (6.9. Fig./b) the two bright states with $\pi$ phase difference can be found at amplitude of 0.55 and the dark state has amplitude of 0.14. In the measurements foil retarders (nominal retardance: 120nm, type IP 120WR01 from International Polariser) were used as quarter wave plates, Polaroid HN38S foil as polariser and analyzer and a 473nm DPSS laser from Crystal Laser. The deviations between the numerical predictions and the measurements can be explained by the same reasons described in point 6.5.1.

80
a) Calculated amplitude and phase modulation characteristics of TN LC cell with $\Gamma=6$ in the configuration: $p=110 \ a=100 \ q1=35 \ q2=45$

b) Measured amplitude and phase modulation characteristics of Sony LCX017DLT TN LCD with 473 nm laser in the configuration: $p=110 \ a=100 \ q1=35 \ q2=45$
6.5.3 Ternary phase-amplitude modulation with $\Gamma=6.6$

In this point the results of a computer search with $\Gamma=6.6$ is shown. This $\Gamma$ value supplies theoretically such bright states that have amplitudes very close to 1, thus very small light loss. The theoretical amplitude and phase modulation characteristics of such a configuration is shown in 6.10. Fig. The bright and dark states have amplitudes of 0.9 and 0.16 respectively. Unfortunately the lack of the availability of lasers with short enough wavelength prevented me to measure this configuration.
6.6 Fourier plane homogenization with test images

In this point the Fourier plane light homogenization is experimentally demonstrated using test images. The measurement setup is shown in 6.11. Fig. The polariser, analyzer and quarter wave plates are set to the configuration described in 2.1. A camera objective (f=50mm) is used to generate the Fourier spectrum and real image of the LCD. The light intensity variation is measured at the back focal plane and the image plane by a CCD camera (Sony XC 73CE). An additional analyzer is used to reduce the light intensity to a level matching to the sensitivity of the CCD.

According to our laboratory measurements the LCD has inhomogeneity along its surface (e.g. LC thickness), thus at different locations of the LCD surface slightly different voltages are required to realize the TPA modulation. All the test images are displayed by 90x90 LCD pixels, since the same voltages can be used to realize TPA modulation within this small area. In order to realize TPA on a larger area of the LCD (1024x768 pixels) the voltage variations can be compensated by appropriate image generation.

The first test image is a chess board (6.12. Fig./a), created by using only two of the TPA states: the maximal (+1 state) and minimal (0 state) voltages (see 6.8. Fig./b). Each bright and dark square is constructed of 3x3 LCD pixels to avoid problems originating from MTF degradation parallel with the LCD rows. In 6.12. Fig./a the black squares represent pixels with maximal and the white squares with minimal voltages, thus the LCD displays a negative image of the original. 6.12. Fig./b shows the intensity variations in the image plane and 6.12. Fig./c shows the intensity in the Fourier plane. 6.13. Fig./a shows a chess board like test image generated by TPA states. The black and gray spots represent the +1 and −1 states and the white spots represent the 0 states. 6.13. Fig./b and 6.13. Fig./c show the intensity in the image and Fourier planes respectively. Comparing 6.12. Fig./c and 6.13. Fig./c it can be seen, that applying TPA modulation the zero order is effectively destructed. Since 6.13. Fig./c shows the Fourier transform of a periodic object, the energy of the zero order is coupled into other orders.
In order to have a continuous intensity distribution in the Fourier plane random image patterns should be applied. 6.14. Fig./a show a random bit pattern created by 1 and 0 states, where black represents 1 states and white represent zero states. 6.14. Fig./b shows the image plane, 6.14. Fig./c shows the Fourier plane and 6.14. Fig./d shows the intensity distribution along a vertical cross section of the Fourier plane. In 6.15. Fig./a the same random bit pattern can be seen using TPA modulation. The dark and gray squares represent the +1 and –1 states, while the white squares represent the 0 states. 6.15. Fig./b and c show the image and Fourier planes applying TPA modulation. 6.15. Fig./d shows the intensity distribution along a vertical cross section of the Fourier plane with TPA modulation. 6.14. Fig./c-d and 6.15. Fig./c-d were measured with exactly the same device settings (laser power, camera integration time, additional analyzer setting, frame grabber settings, etc.). Comparing the intensity images of the two Fourier spectra (6.14. Fig./c-d and 6.15. Fig./c-d) it can be seen, that TPA
modulation has a “more homogenous” Fourier spectrum than conventional intensity modulation. In the case of conventional intensity modulation (6.14. Fig./c-d) there is a strong peak at zero frequency. Since in 6.14. Fig./c the camera was saturated at the peak, the peak per average intensity ratio can not be measured from this image. In order to overcome the dynamic range limitations of the camera images with smaller laser powers were taken, in which the CCD was not saturated in the peak. By comparing two images with different laser powers (one with not saturated peak and one with saturated peak) the peak per average ratio could be measured. The average intensity was calculated for pixels located inside the Nyquist aperture belonging to a square of 3x3 SLM pixels (see 6.14. Fig./c). The obtained peak per average intensity ratio was 205:1. By applying TPA the zero frequency component is destructed and the energy is more homogenously distributed in the whole frequency plane (6.15. Fig./c-d). Assuming that the CCD has a linear response the peak per average intensity ratio can be measured from 6.15. Fig./c since the image is not saturated at the peak. This measurement yields an intensity ratio of 8:1 between peak and average. The average intensity was calculated inside the Nyquist aperture of 3x3 SLM pixels also in this case. The Nyquist aperture is worth to be stored in Fourier holography since it contains more than 80% of the total energy. The peak per average intensity ratio of the hologram is a very important parameter, since it determines the required dynamic range of the storing material.
a) Random bit image using conventional intensity modulation. Black squares: +1, white squares: 0

b) Measured intensity distribution in image plane for image a

c) Measured intensity distribution in Fourier plane for image a, Nyquist aperture of 3x3 SLM pixels is shown

d) Intensity profile along a cross section of Fourier plane for conventional intensity modulation
a) Random bit image using TPA modulation. Black squares: +1, gray squares: -1, white squares: 0

b) Measured intensity distribution in image plane for image a

c) Measured intensity distribution in Fourier plane for image a, Nyquist aperture of 3x3 SLM pixels is shown

d) Intensity profile along a cross section of Fourier plane for TPA modulation

6.15. Fig.
6.7 Summary and conclusions

In this chapter I demonstrated the effective realization of TPA modulation with twisted nematic LCDs. LCDs with different \( \Gamma \) were analyzed. The optimal illuminated and detected elliptical polarizations were determined by computer search applying the Jones matrix model of TN LCD. Different elliptical polarizations were proposed for phase retardations of \( \Gamma = 5.2 \), \( \Gamma = 6 \) and \( \Gamma = 6.6 \), that theoretically realized TPA modulation with light amplitudes of 0.5, 0.6 and 0.9 respectively. The calculated intensity contrast between the bright and dark states were in all cases better than 15. The two configurations proposed for \( \Gamma = 5.2 \) and \( \Gamma = 6 \) were experimentally tested. In both configurations TPA modulation could be realized but the measured amplitude of the bright states were about 10% smaller than the calculated amplitudes. The measured intensity contrast between bright and dark states were in both configurations around 15. The intensity smoothing in the Fourier plane is also demonstrated for the configuration of \( \Gamma = 5.2 \) by applying test images. The computer searches resulted a huge number of different configurations realizing TPA, which were not evaluated. Probably other configurations would realize TPA with similar or even better characteristics. The intended application area of our results is holographic data storage, however they can be also applied for realizing TPA correlation filters.
7 Appendix I

The eye safety analysis of the light source based parabolic mirror and LED array according to the European Norm EN-60825-1/A11

For the analysis of the eye safety of the light source based on cylindrical parabolic mirror and LED array I use the German version of the European Norm E DIN 60825-1/A11 accepted in 1996 in Germany. This standard contains a base standard (E DIN 60825-1) from 1994 and an extension (A11) from 1996. The standard relates to the safety of laser sources, but the term laser includes sources constructed of LEDs as well. In the article describing the stripe source with parabolic mirror and LED array I concluded that the light source belongs to the Class III/B, namely direct looking into the light source is dangerous but viewing into diffuse reflection is safe. This classification is far too strict it was based on a misinterpretation of the standard. In this appendix I will show that the exposure of the light source is far below the Maximal Permissible Exposure (MPE) defined by the standard. The MPE is defined as the maximal exposure to the eye and skin without any short or long term hazard of injury. In point 4.5.2.1 was described that the parabolic mirror creates two virtual quasi-images about the LED array, one at distance f behind the mirror and one at infinity. The virtual image at distance f behind the mirror was found to be more dangerous so only this “apparent” source will be examined.

Measurement setup for eye safety of laser sources

7.1. Fig.

Die MZB-Werte (in English MPE, my addition) stellen die maximalen Werte dar, denen die Auge oder die Haut ausgesetzt werden können, ohne dass damit Verletzungen unmittelbar oder nach einer langen Zeit verbunden sind." See point 3.51 in [4].
The principal measurement setup for evaluating the eye safety of the source specified by the norm is shown in 7.1. Fig. It contains an aperture of 7mm diameter placed at the smallest possible distance from the apparent source, but not closer than 100mm. It also contains a lens that images a subsection of the apparent source inside a cone of angle \( \alpha \). Parameter \( \alpha \) is the angular extension of the apparent source measured at the smallest possible distance, but not closer than 100mm. If the apparent source is of rectangular shape, \( \alpha \) is defined as the average of the angular extensions measured along the two sides of the rectangle with the conditions that any angle smaller than \( \alpha_{\text{min}} \) or bigger than \( \alpha_{\text{max}} \) must be replaced by \( \alpha_{\text{min}} \) or \( \alpha_{\text{max}} \) respectively. \( \alpha_{\text{max}} \) is equal to 0.1 radian for all cases while \( \alpha_{\text{min}} \) is a function of the exposure time.
To measure the exposure I used the optical setup shown in 7.2. Fig. and 7.3. Fig. The apparent source is in this case the virtual images of the LED chips at distance f behind the lens. The focal length of the cylindrical parabolic mirror f is equal to 40mm, it can be calculated from its equation (y =1/(2*80mm)*x^2 [mm] see the table caption of 4.1. Table). A lens of 7mm diameter images subsection of the virtual images of the LED chips onto a photo detector. The subsection imaged onto the detector is defined by a cone having an apex at the centre of the lens and an angle of α_{max}=0.1 radian. The lens is placed into 100mm distance from the virtual image of the LED chip array as specified by the standard, and its optical axis is directed parallel with axis z.

The apparent source is a complex extended source and the norm specifies that for such sources not only the whole source but all subsections of the source should be examined in order to decide about the eye safety. As there are infinite number of subsections and all of them can not be evaluated, so I will examine only two subsections: one is the virtual image created by a single LED chip and the other is the whole virtual image. It can be shown by simple considerations that one of these two subsections are the most dangerous according to the standard.

The light source illuminates 0.04s long impulses with 5 Hz repetition frequency. For such an impulse modulated illumination the norm prescribes three different examinations.

A. The exposure of a single impulse should be below the specified MPE.
B. The sum of the light exposure illuminated by the pulse train in time T should be less than the MPE allowed for a single pulse of length T. T is equal to 100sec for our source.
C. The single pulse exposure should be compared with the MPE of the single pulse multiplied with a correction factor C as defined below:
   \[ C_6 = N^{1/4} \]
   Where N is the number of impulses in 100sec.

In the remaining part of this appendix the MPEs for the two subsections and for the three above cases will be calculated according to the norm and they will be compared with measured value.

### 7.1 Virtual image of a single LED chip

The first examined subsection of the complete light source is the virtual image of a single LED (see 7.3. Fig.). The MPE for the single pulse originating from a single LED can be calculated on the following way:

\[
MPE = 18 \cdot t^{0.75} \cdot C_4 \cdot \frac{J}{m^2}
\]

7.1. Eq.

Where

- t: illumination time, 0.04s
- \(C_4 = 10^{0.002(\lambda-700)}\): for \(\lambda=880nm\) \(C_4=2.29\)
- \(C_6 = \alpha/\alpha_{\min}\)
- \(\alpha::\) is the angular extension of the source
\( \alpha_{\text{max}} = 0.1 \text{ rad} \)
\( \alpha_{\text{min}} = 0.0015 \text{ rad} \)

\( \alpha \) is specified as the average value of the angular extensions of the apparent source in \( x \) and \( y \) directions applying the condition that any angular size smaller than \( \alpha_{\text{min}} \) or greater than \( \alpha_{\text{max}} \) should be replaced by \( \alpha_{\text{min}} \) and \( \alpha_{\text{max}} \) respectively. The angular size of the virtual image of a single LED chip in direction \( x \) is \( \alpha_x = 0.4/100 = 0.004 \text{ rad} \) and in direction \( y \) \( \alpha_y = 60/100 = 0.6 \text{ rad} \) but because 0.6 is bigger than \( \alpha_{\text{max}} = 0.1 \) so \( \alpha_y = 0.1 \) (see 7.3. Fig.). The angular size \( \alpha \) is the average of \( \alpha_x \) and \( \alpha_y \) so \( \alpha = (0.004 + 0.1)/2 = 0.052 \). Using these values the MPE of single impulse can be obtained by the straightforward calculations:

\[ \text{MPE}^A = 18 \cdot 0.04^{0.75} \cdot 2.29 \cdot \frac{0.052}{0.0015} \text{ J/m}^2 = 128 \text{ J/m}^2 \]

The correction factor \( C_5 \) is \( C_5 = 500^{-1/4} = 0.21 \) as the number of impulses repeated with 5Hz frequency in 100sec is 500. So the MPE of a single impulse of 0.04sec should be multiplied with \( C_5 \) to obtain the modified MPE:

\[ \text{MPE}^C = 128 \cdot \frac{\text{J}}{\text{m}^2} \cdot C_5 = 27 \text{ J/m}^2 \]

For calculating the MPE of an impulse of 100sec the angular size of the source should be recalculated since \( \alpha_{\text{min}} = 0.011 \text{ rad} \) in this case. So the correct angular extensions in this case are: \( \alpha_x = \alpha_{\text{min}} = 0.011 \) and \( \alpha_y = \alpha_{\text{max}} = 0.1 \). The average of \( \alpha_x \) and \( \alpha_y \) gives \( \alpha \) namely \( \alpha = (0.011 + 0.1)/2 = 0.055 \). The MPE of a 100sec long pulse can be calculated by

\[ \text{MPE} = 18 \cdot 100^{0.75} \cdot 2.29 \cdot \frac{0.055}{0.011} \text{ J/m}^2 = 6517 \text{ J/m}^2 \]

Since there are 500 pulses in the 10sec interval the MPE of a single pulse is the following:

\[ \text{MPE}^B = \frac{6517 \text{ J}}{500 \text{ m}^2} = 13 \text{ J/m}^2 \]

Comparing MPE\(^A\), MPE\(^B\) and MPE\(^C\) it can be seen that the most restrictive of them is MPE\(^A\).

The exposure of a single LED was measured according to 7.2. Fig. and 7.3. Fig. by blocking the radiation of all LEDs except one. The measured exposure was:

\[ E_{\text{MEAS}} = 0.16 \text{ J/m}^2 \]

The above value shows that the one LED of the light source is eye safe.

### 7.2 The virtual image of the whole LED array

If the virtual image of the whole LED array is regarded as an extended source then the angular size of the source is equal to \( \alpha_{\text{max}} \), namely \( \alpha = \alpha_{\text{max}} = 0.1 \text{ rad} \) (see 7.3. Fig.). The MPE of a single pulse of 0.04sec length is given by the following formula applying 7.1. Eq.
\[ \text{MPE}^A = 18 \cdot 0.04^{0.75} \cdot 2.29 \cdot \frac{0.1}{0.0015} \frac{\text{J}}{\text{m}^2} = 245 \frac{\text{J}}{\text{m}^2} \]

The MPE modified by the correction factor \( C_5 = 0.21 \) originating from the train of 500 impulses in 100 sec is:

\[ \text{MPE}^C = 245 \cdot C_5 \frac{\text{J}}{\text{m}^2} = 52 \frac{\text{J}}{\text{m}^2} \]

The MPE of a 100 sec impulse according to 7.1. Eq. and applying \( \alpha_{\text{min}} = 0.011 \) is the following:

\[ \text{MPE} = 18 \cdot 100^{0.75} \cdot 2.29 \cdot \frac{0.1}{0.011} \frac{\text{J}}{\text{m}^2} = 11850 \frac{\text{J}}{\text{m}^2} \]

The MPE of a single impulse of 0.04 sec can be obtained by dividing this value by 500, as follows:

\[ \text{MPE}^B = \frac{11850}{500} \frac{\text{J}}{\text{m}^2} = 23.7 \frac{\text{J}}{\text{m}^2} \]

Comparing the three MPE values it can be easily seen that even in this case the most restrictive of them is \( \text{MPE}^B \).

The exposure of the whole light source was also measured according to 7.2. Fig. and 7.3. Fig. and it was found to be:

\[ E_{\text{MEAS}} = 0.36 \frac{\text{J}}{\text{m}^2} \]

The measured exposure is far below \( \text{MPE}^B \) thus the whole light source is also eye safe.
8 Appendix II

The result that is proven in this appendix is already known, however the steps of the proof are omitted from the literature [31.][32.] I read. I found necessary to make this proof to show that there is no further neglected phase modulation factors within it.

In point 6.2 it was found that the Jones matrix describing the polarisation and phase modulation of a TN LC cell contains the following matrix power (see 6.6. Eq.):

\[
\begin{bmatrix}
\cos\left(\frac{\alpha}{N}\right)e^{-j\frac{\Gamma}{2N}} & \sin\left(\frac{\alpha}{N}\right)e^{-j\frac{\Gamma}{2N}} \\
-sin\left(\frac{\alpha}{N}\right)e^{j\frac{\Gamma}{2N}} & \cos\left(\frac{\alpha}{N}\right)e^{j\frac{\Gamma}{2N}}
\end{bmatrix}^N
\]

8.1. Eq.

This matrix can be further simplified using Chebyshev’s identity for unimodular matrices:

\[
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix}^N = \begin{bmatrix}
A\cdot\sin(N\cdot Z) - \sin((N-1)\cdot Z) & B\cdot\frac{\sin(N\cdot Z)}{\sin(Z)} \\
C\cdot\frac{\sin(N\cdot Z)}{\sin(Z)} & D\cdot\sin(N\cdot Z) - \sin((N-1)\cdot Z)
\end{bmatrix}
\]

\[
Z = \arccos\left(\frac{1}{2}\cdot(A + D)\right)
\]

8.2. Eq.

The limit of 8.1. Eq. when N tends to infinite (N→∞) will be calculated. The parameter Z of 8.2. Eq. can be expressed using 8.1. Eq. as

\[
\cos(Z) = \frac{1}{2}\left(\cos\left(\frac{\alpha}{N}\right)e^{-j\frac{\Gamma}{2N}} + \cos\left(\frac{\alpha}{N}\right)e^{j\frac{\Gamma}{2N}}\right) = \cos\left(\frac{\alpha}{N}\right)\cdot\cos\left(\frac{\Gamma}{2\cdot N}\right)
\]

8.3. Eq.

Applying the second order Taylor approximation for both sides of 8.3. Eq. and neglecting the element proportional to 1/N^4 we obtain

\[
1 - \frac{Z^2}{2} = \left(1 - \frac{\alpha^2}{2\cdot N^2}\right)\left(1 - \frac{\Gamma^2}{8\cdot N^2}\right) \approx 1 - \frac{\alpha^2}{2\cdot N^2} - \frac{\Gamma^2}{8\cdot N^2}
\]

\[
Z = \sqrt{\frac{\alpha^2}{N^2} + \frac{\Gamma^2}{4\cdot N^2}}
\]

8.4. Eq.
The element (1,2) of 8.2. Eq. using 8.1. Eq. can be expressed as follows:

\[
B \cdot \frac{\sin(N \cdot Z)}{\sin(Z)} = \sin\left(\frac{\alpha}{N}\right) \cdot e^{-\frac{\Gamma}{2N}} \cdot \frac{\sin(N \cdot Z)}{\sin(Z)}
\]

When \(N \to \infty\) the exponential factor of the above equation tends to 1 \((e^{\frac{\Gamma}{2N}} \to 1)\) so it can be omitted.

Using 8.4. Eq. the above equation can be expressed as

\[
\sin\left(\frac{\alpha}{N}\right) \cdot \frac{\sin\left(N \cdot \sqrt{\frac{\alpha^2 + \Gamma^2}{4 \cdot N^2}}\right)}{\sin\left(\frac{1}{N} \cdot \sqrt{\frac{\alpha^2 + \Gamma^2}{4}}\right)} = \frac{\sin\left(\frac{\alpha}{N}\right)}{\sqrt{\frac{\alpha^2 + \Gamma^2}{4}}}
\]

The limit of the above equation when \(N\) tends to infinity thus \(1/N\) tends to zero \((1/N \to 0)\) can be easily obtained using L’Hospital’s rule\(^*:\)

\[
B \cdot \frac{\sin(N \cdot Z)}{\sin(Z)} \xrightarrow{N \to \infty} \frac{\alpha}{\sqrt{\alpha^2 + \frac{\Gamma^2}{4}}}
\]

Let’s introduce a new parameter \(X\) as it is defined below:

\[
X = \sqrt{\alpha^2 + \frac{\Gamma^2}{4}}
\]

8.5. Eq.

The element (1,2) of the result matrix can be written in the following form applying 8.5. Eq.:

\[
B \cdot \frac{\sin(N \cdot Z)}{\sin(Z)} \xrightarrow{N \to \infty} \frac{\alpha}{\sqrt{\alpha^2 + \frac{\Gamma^2}{4}}} \cdot \frac{\sin(X)}{X}
\]

8.6. Eq.

When \(N \to \infty\) \(C = -B\) is true for 8.1. Eq. \((C\) is defined in 8.2. Eq.\) so element (2,1) of the result matrix after power \(N\) can be simply obtained from 8.7. Eq.:

\[
C \cdot \frac{\sin(N \cdot Z)}{\sin(Z)} \xrightarrow{N \to \infty} -\frac{\alpha}{\sqrt{\alpha^2 + \frac{\Gamma^2}{4}}} \cdot \frac{\sin(X)}{X}
\]

8.8. Eq.

\(^{*}\text{According to the L’Hospital’s rule the limit of the ratio of two sinus functions when the arguments of both tend to zero can be calculated on the way shown below:}\)

\[
\frac{\sin(a \cdot X)}{\sin(b \cdot X)} \xrightarrow{x \to 0} \frac{a \cdot \cos(a \cdot X)}{b \cdot \cos(b \cdot X)} = \frac{a}{b}
\]
The element \((1,1)\) of the result matrix can be written after some trigonometric changes using Eq. 8.1 and 8.2. As

\[
A \cdot \sin(N \cdot Z) - \sin((N - 1) \cdot Z) = \frac{A \cdot \sin(N \cdot Z) - \sin(N \cdot Z) \cdot \cos(Z) + \cos(N \cdot Z) \cdot \sin(Z)}{\sin(Z)} = \frac{\sin(N \cdot Z) \cdot (A - \cos(Z))}{\sin(Z)} + \cos(N \cdot Z)
\]

Substituting Eq. 8.2 into the above equation yields:

\[
\frac{\sin(N \cdot Z) \cdot (A - \cos(Z))}{\sin(Z)} + \cos(N \cdot Z) = \frac{\sin(N \cdot Z) \cdot \frac{1}{2} (A - D)}{\sin(Z)} + \cos(N \cdot Z)
\]

Substituting \(A - D\) from Eq. 8.1 into the above equation results:

\[
\frac{\sin(N \cdot Z) \cdot \frac{1}{2} (A - D)}{\sin(Z)} + \cos(N \cdot Z) = \frac{-\sin(N \cdot Z) \cdot j \cdot \cos \left( \frac{\alpha}{N} \right) \cdot \sin \left( \frac{\Gamma}{2 \cdot N} \right)}{\sin(Z)} + \cos(N \cdot Z)
\]

In the above equation \(\cos(\alpha/N)\) tends to 1 when \(N \to \infty\) so it can be eliminated. The substitution of \(Z\) from Eq. 8.4 into the above equation yields:

\[
- \frac{j \cdot \sin \left( \sqrt{\alpha^2 + \Gamma^2} \right)}{\sin \left( \frac{1}{N} \sqrt{\alpha^2 + \Gamma^2} \right)} + \cos \left( \sqrt{\alpha^2 + \Gamma^2} \right)
\]

The limit of the above formula when \(N \to \infty\) can be obtained applying L’Hospital’s rule:

\[
\lim_{N \to \infty} \frac{A \cdot \sin(N \cdot Z) - \sin((N - 1) \cdot Z)}{\sin(Z)} = -\frac{j \cdot \sin \left( \sqrt{\alpha^2 + \Gamma^2} \right)}{\sin \left( \sqrt{\alpha^2 + \Gamma^2} \right)} \cdot \frac{\Gamma}{\sqrt{\alpha^2 + \Gamma^2}^2} + \cos \left( \sqrt{\alpha^2 + \Gamma^2} \right)
\]

Using parameter \(X\) (defined by Eq. 8.6) Eq. 8.9 can be rewritten into the form shown below:

\[
\lim_{N \to \infty} \frac{A \cdot \sin(N \cdot Z) - \sin((N - 1) \cdot Z)}{\sin(Z)} = -\frac{j \cdot \Gamma \cdot \sin(X)}{X} + \cos(X)
\]
8.10. Eq. gives the element (1,1) of the result matrix after power N. When $N \to \infty$ $A = D$ is true for 8.1. Eq. (A, D are defined in 8.2. Eq.) so element (2,2) of the result matrix after power N is also given by 8.10. Eq. As all the four elements of the result matrix are known we can define it applying 8.7. Eq., 8.8. Eq. and 8.10. Eq.:

$$
\begin{bmatrix}
\cos\left(\frac{\alpha}{N}\right)e^{-\frac{j\Gamma}{2N}} & \sin\left(\frac{\alpha}{N}\right)e^{-\frac{j\Gamma}{2N}} \\
-\sin\left(\frac{\alpha}{N}\right)e^{\frac{j\Gamma}{2N}} & \cos\left(\frac{\alpha}{N}\right)e^{\frac{j\Gamma}{2N}}
\end{bmatrix}^N = 
\begin{bmatrix}
\cos(X) - j \cdot \frac{\Gamma}{2} \cdot \frac{\sin(X)}{X} & \alpha \cdot \frac{\sin(X)}{X} \\
-\alpha \cdot \frac{\sin(X)}{X} & \cos(X) + j \cdot \frac{\Gamma}{2} \cdot \frac{\sin(X)}{X}
\end{bmatrix}
$$

8.11. Eq.
9 References


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