Fluctuation Phenomena on the Stock Market

Ph.D. thesis

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1. Background

During the last decade physicists have published dozens of books and thousands of scientific papers in the field of finance. This new trend – like many others – stems from the huge development of statistical physics that started in the 70’s. In this period many new concepts and models were born, such as fractal and multifractal scaling, frustrated disordered systems, or strongly non-equilibrium phenomena and the tools necessary for their description.

Inspired by these, studies in ”econophysics” concentrate on possible analogies between financial and physical systems, but there is a large discrepancy in the level of understanding between the two fields. For example, in the case of a physical system undergoing a second order phase transition the assumption of scaling and the determination of universal exponents is a well motivated undertaking. In the behavior of financial markets there is no basis whatsoever for a similar treatment, and the assumption of power laws is just one of many possibilities for the description of empirical data. Universality, as present in physical systems, is also a much stronger property, than the existence of ”stylized facts”, the qualitative similarity of stock market data throughout various stocks and markets.

The general observations in stock markets include the broad, non-normal distribution of returns (change of the logarithm of price), the absence of linear autocorrelations in returns, and the long memory of volatility (time dependent standard deviation of returns). The trading activity of stocks (expressed in value or
number of shares) is also characterized by a broad distribution and strong temporal autocorrelations.

Although these facts are well known phenomenologically, their accurate description and origin remains an important field of research even today. The most plausible reason for such interest is of course that by devising the appropriate pricing models and trading strategies, it is possible to obtain substantial speculative profits in financial markets. Investment firms often employ physicists in the related areas.

There are, however, further reasons too. The market can be understood as a self-adaptive complex system whose operation is determined both by external influences and a complicated internal structure. The former is comprised by news and macroeconomic effects, while the latter by countless interconnected, interacting firms, banks, brokers. The understanding of the dynamics of market investment decisions could lead the way to significant improvements: The structure and legislation of markets could be changed in the hope of a more efficient operation. For example, it might become possible to dampen the fluctuations and the effects of bubbles that cause market crashes and potentially significant economic damage.

2. Goals

The most important goal of my research was to critically revisit the stylized facts regarding the dynamics of trading, and then describe these phenomena by scaling theories and random models. Besides this, I have put great emphasis on facilitating a
stronger connection between the theory of complex systems and the description of markets. This was achieved through the phenomenon of fluctuation scaling and the introduction of a unified scaling theory. This theory and my empirical results also made it inevitable to revise several statements found in the recent literature.

The notion of stock market fluctuations can be approached empirically in two ways. One is based on the transaction data characterizing the trading activity of the whole market. For this study I used the TAQ database containing all transactions on the New York Stock Exchange and NASDAQ in the period 1993-2003. The other approach is the analysis of the so called limit order book that contains all individual buy and sell offers. The order book is the most complete source of information available to the participants of trading. It makes possible a very detailed inquiry into the nature of price formation, the trading process and market microstructure. My aim was to identify the most important features of the dynamics of the order book on different time scales. Based on these it became possible to analyze, understand and model the statistics of individual order executions. For this research I used the complete order book data of the London Stock Exchange for the year 2002.

3. Methods

My work included filtering and processing the above mentioned databases, their statistical analysis and comparison with Monte Carlo simulations. For the interpretation of the results it was
necessary to create, analyze and simulate benchmark models. For the solution of the great majority of these tasks I used the programming language C. My theoretical calculations were based on tools of probability theory, the theory of random walks, and a phenomenological scaling theory.

4. New scientific results

1. Many complex systems are characterized by the fluctuation scaling relationship between its $i$ elements. This means that for some positive, additive, time-dependent quantity $f_i(t)$ the standard deviation $\sigma_i$ and the mean $\langle f_i \rangle$ is related by the law

$$\sigma_i \propto \langle f_i \rangle^\alpha.$$

I have shown that if one chooses $f_i(t)$ as the traded value of the $i$'th share on the stock market during the period $[t, t + \Delta t)$ the above scaling law is valid, and $\alpha$ strongly depends on the window size $\Delta t$. [1]

2. I introduced a combined scaling theory of long-range temporal autocorrelations and fluctuation scaling. In this framework I showed that the behavior of the $\alpha(\Delta t \to 0)$ limit is explained by the relationship of the frequency and the size of the transactions. Furthermore, I showed that the time window dependence of $\alpha$ is equivalent to that for short times trading activity is uncorrelated, and for longer times it is characterized by the $H_i = H_* + \gamma \log \langle f_i \rangle$ Hurst exponents. Since $\langle f \rangle$ essentially characterizes liquidity, this
logarithmic relationship is proof, that the strength of correlations displays systematic non-universality. [2, 3, 5–7]

3. I extended fluctuation scaling and the related scaling theory to the analysis of higher moments. In the generalized, multiscaling form

\[ \langle |f_i - \langle f_i \rangle|^q \rangle \propto \langle f_i \rangle^{\alpha(q)}. \]

I showed that this expression can be applied to the distribution of stock market traded value. I also showed that the correlation properties of \( f \) also display multiscaling, and the \( \tau(q) \) scaling function and the multifractal spectrum are both liquidity dependent. [1, 8]

4. By the comparative application of various fitting methods I showed, that the distribution of the \( f \) traded value, in contrast with earlier statements in the literature, has a finite second moment, and thus the related Hurst exponent is well defined. I determined that the correlations of traded value have two origins: correlations between the \( V \) size of consecutive transactions, and in the \( N \) number of trades in the given time interval. The time dependence of \( V \) and \( N \) is characterized by non-universal Hurst exponents, and both give a significant contribution to the correlations of \( f \). [4, 8, 9]

5. I showed, that order book level data display different qualitative behavior on different time scales. The distributions of time to fill and time to cancel decay asymptotically as
power laws with the same exponent $\lambda_{TTF} \approx \lambda_{TTC} \approx 2$. This value is greater than the $\lambda_{FPT} \approx 1.5$ value characterizing the first passage time of price. Based on a random walker model I showed that the order lifetimes are also asymptotically power law distributed, and the related exponent can be expressed as $\lambda_{LT} = \lambda_{TTF} - \lambda_{FPT} + 1$. This relationship can be used to calculate $\lambda_{LT}$ in the case of real data where it cannot be observed directly. I investigated the dependence of the above processes as the function of a $\Delta$ distance from the best price. Based on Monte Carlo simulations I showed that the results of the model do not change significantly if one relaxes the simplifying assumptions. [10, 11]

5. Scientific publications related to the thesis


5 SCIENTIFIC PUBLICATIONS RELATED . . .


6. Further scientific publications


