Thesis booklet

Investigation of 2D hybrid nanostructures

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Introduction

The semiconductor industry is approaching a limit where Moore's law [1] is becoming increasingly difficult to sustain [2]. Alternative approaches building on the results of quantum mechanics are required to further enhance the functionality of electronic devices. For example, recent years have seen a surge in encouraging achievements in the field of quantum computation. Most notably, the quantum advantage over classical computers for specific problems was demonstrated using superconducting [3, 4] and photonic [5] quantum computers. Superconducting qubits are among the most promising platforms to create scalable and programmable quantum computers capable of solving practical problems. Furthermore, superconducting hybrid devices are also proposed to host exotic quasiparticles such as Majorana fermions[6–8] that may enable fault-tolerant quantum computing [9]. Majorana fermions are expected to arise if superconducting correlations are induced in the surface states of topological insulators [7].

Graphene has been theoretically predicted as a topological insulator soon after its discovery in 2004 [10]. However, the experimental observation of this exotic phase in graphene has remained elusive due to its very weak intrinsic spin–orbit coupling (SOC) [11]. On the other hand, the family of two-dimensional (2D) materials has grown rapidly over the last two decades, making it possible to tailor the physical porperties of graphene by creating van der Waals heterostructures that combine graphene and other 2D materials. For example, by bringing graphene in close proximity to transition metal dichalcogenides (TMDs) in such heterostructures, a large SOC can be induced in graphene [12]. This, on the one hand, gave a significant boost to the field of spintronics. The combination of the large spin diffusion length in graphene [13–15] and the ability to manipulate spins by electric fields [16–18] are key elements to realize information storage and logic devices that utilize the spins of electrons [19]. On the other hand, this so-called proximity-induced SOC has opened new possibilities to engineer topological phases in graphene [20–22], leading to the experimental observation of a peculiar band-inverted phase hosting helical edge states in bilayer graphene [23, 24]. Furthermore, the induced SOC can also have a strong effect on the correlated states observed in twisted structures [25–27].

Objectives

Proximity-induced spin–orbit coupling in graphene

By combining graphene with other 2D materials that have a large intrinsic spin–orbit coupling, it becomes possible to induce a significant SOC in graphene via the proximity-effect [12, 28–31]. Although other methods have also been proposed to enhance SOC in graphene [19, 32, 33], TMDs are among the most promising candidates to enable spintronics applications in graphene devices. It was found both theoretically [29] and experimentally [12, 31] that graphene/TMD heterostructures make it possible to engineer a large proximity-induced SOC in graphene while preserving its high electronic quality. Among the family of TMD materials, WS_2 [12, 34], MoS_2 [34] and WSe_2 [34, 35] have all been demonstrated to induce a SOC in graphene on the order of $\sim 1-10$ meV that is multiple orders of magnitude larger than the intrinsic SOC in pristine single-layer graphene.

I fabricated van der Waals heterostructures that combine single- and bilayer graphene with WSe_2. These heterostructures allow the investigation of proximity-induced SOC in graphene using low-temperature transport measurements. Our research group has developed a method to perform transport measurements on van der Waals heterostructures under hydrostatic pressure. This method is applied to boost the proximity-induced SOC in a bilayer graphene/WSe_2 heterostructure. Using low temperature transport measurements, I investigated the effect of hydrostatic pressure on the band-inverted phase arising from the proximity-induced SOC.

Current–phase relation measurements of WSe_2/graphene heterostructures

Josephson junctions, consisting of two superconducting leads connected by a weak link, are the building blocks of state-of-the-art superconducting qubits [3, 4]. The current–phase relation (CPR) is the most fundamental property of a Josephson junction. It relates the magnitude of the dissipationless supercurrent in the weak link to the macroscopic phase difference of the two superconducting leads. Therefore, it provides information on the physical process underlying the supercurrent transport in the junction. Furthermore, it is also expected to be affected by SOC in the weak link [36] that can give rise to anomalous Josephson effect, resulting in the appearance of a phase shift in the CPR [37–40] and the superconducting diode effect that manifests in the asymmetry of the critical current, the maximum allowed supercurrent in the weak link, for different current directions [40–48].

I measured the current–phase relation of graphene/WSe_2 heterostructures using two Josephson junctions in an asymmetric SQUID configuration. In such devices, the loop inductance can have a significant impact on the measured CPR. Taking this into account, I investigated the phase shifts of the CPR in in-plane magnetic fields and the limitations of this method to determine small phase shifts.

Multiterminal Josephson junctions

Multiterminal Josephson junctions (MTJJs) consisting of a single scattering region connected to multiple superconducting terminals attracted significant attention in recent years. Theoretical works showed that MTJJs may enable multiplet supercurrents [49], and the Andreev bound state (ABS) spectra of MTJJs can exhibit non-trivial topology and simulate the band structure of Weyl semimetals [50].

I investigated a graphene-based three-terminal Josephson junction using low-temperature transport measurements. I investigated the effect of self-heating due to the coexistence of normal current and supercurrent in these devices. The behavior of such devices can be described by a network of resistively shunted Josephson junctions. I showed that simulations can be further improved if these self-heating effects are taken into account. The switching dynamics of the device were also probed by the measurement of its switching current distribution.

Thesis points

1. I created van der Waals heterostructures based on single-layer graphene, hexagonal boron nitride (hBN), and tungsten diselenide (WSe_2) to induce spin—orbit coupling in graphene. One of these devices allowed our coworkers to investigate the spin relaxation times related to the spin—orbit coupling induced by WSe_2 in graphene

as a function of momentum relaxation time, which enabled the identification of the relevant spin relaxation mechanism and the large spin-relaxation anisotropy. These results are presented in [T1]. I also fabricated a heterostructure consisting of single-layer graphene and hBN that enabled our research group to test if hBN can protect graphene from kerosene used in a pressure cell. This result is published in [T2]. Furthermore, in [T3], using low-temperature transport measurements, I showed that the band-inverted phase formed in bilayer graphene due to double-sided WSe_2 encapsulation can be stabilized using hydrostatic pressure. By performing activation measurements on this WSe_2/bilayer graphene/WSe_2 heterostructure, I determined the magnitude of the induced spin-orbit coupling with and without hydrostatic pressure. I also confirmed the increase of the spin—orbit coupling strength in the heterostructure due to hydrostatic pressure by measuring Landau level crossing points. [T3]

2. I fabricated superconducting quantum interference devices (SQUIDs) from Josephson junctions based on heterostructures containing single-layer and bilayer graphene, hBN, and WSe_2, which allowed me to perform current-phase relation (CPR) measurements. In the case of a Josephson junction containing a WSe_2/single-layer graphene/WSe_2 heterostructure, I showed by CPR measurements that resistance oscillations caused by ballistic Fabry-Perot (FP) interference are also detectable in the superconducting critical current. Furthermore, I demonstrated with these measurements that the skewness of the CPR is enhanced at high doping, indicating high transparency of the conduction channels. Moreover, I have shown that the p-n junctions formed in the junction that also led to the formation of the FP oscillations, led to decreased skewness in the bipolar regime. Additionally, I investigated the phase shifts of the current—phase relation in an in-plane magnetic field. By increasing the magnetic field, I showed phase shifts that cannot be explained by imperfect sample orientation or inductive effects. In connection with these measurements, I demonstrated the practical limitations of the measurement of phase shifts. [T4]

3. I fabricated three-terminal Josephson junctions based on graphene and hBN and performed low-temperature transport measurements on a device. I showed that the behavior of these samples can be described in the measurements by a network model containing three resistively shunted Josephson junctions. In connection with the model, I showed that a more accurate agreement with measurements can be achieved if self-heating effects due to normal currents are also taken into account using electron-phonon coupling. I investigated the behavior of the switching current distribution of the three-terminal Josephson junction. Using this, I showed that its switching dynamics are governed by phase diffusion when the entire sample is in the superconducting state. Furthermore, I showed that if supercurrents and normal currents coexist in the sample, the switching dynamics change, and the damping increases due to the increased temperature. [T5]

List of publications

T1 Simon Zihlmann, Aron W. Cummings, José H. Garcia, Máté Kedves, Kenji Watanabe, Takashi Taniguchi, Christian Schönenberger, and Péter Makk, Large spin re-

laxation anisotropy and valley-Zeeman spin-orbit coupling in WSe_2/Gr/hBN heterostructures, Phys. Rev. B 97, 075434 (2018)

T2 Bálint Fülöp, Albin Márffy, Endre Tóvári, Máté Kedves, Simon Zihlmann, David Indolese, Zoltán Kovács-Krausz, Kenji Watanabe, Takashi Taniguchi, Christian Schönenberger, István Kézsmárki, Péter Makk, and Szabolcs Csonka. New method of transport measurements on van der Waals heterostructures under pressure, J. Appl. Phys, 130(6):064303 (2021)

T3 Máté Kedves, Bálint Szentpéteri, Albin Márffy, Endre Tóvári, Nikos Papadopoulos, Pra-sanna K. Rout, Kenji Watanabe, Takashi Taniguchi, Srijit Goswami, Szabolcs Csonka, and Péter Makk, Stabilizing the inverted phase of a WSe_2/BLG/WSe_2 heterostructure via hydrostatic pressure, Nano Letters 23 (20), 9508-9514 (2023)

T4 Máté Kedves, Prasanna K. Rout, Nikos Papadopoulos, Kenji Watanabe, Takashi Tanigu-chi, Szabolcs Csonka, Srijit Goswami, Péter Makk, Current–phase relation measurements of WSe_2/graphene heterostructures, *manuscript under preparation*

T5 Máté Kedves, Tamás Pápai, Gergő Fülöp, Kenji Watanabe, Takashi Taniguchi, Péter Makk, and Szabolcs Csonka, Self-heating effects and switching dynamics in graphene multiterminal Josephson junctions, Phys. Rev. Research 6, 033143 (2024)

Other unrelated publications

T6 Simon Zihlmann, Péter Makk, Mirko K. Rehmann, Lujun Wang, Máté Kedves, David Indolese, Kenji Watanabe, Takashi Taniguchi, Dominik M. Zumbühl, and Christian Schönen-berger, Out-of-plane corrugations in graphene based van der Waals heterostructures, Phys. Rev. B 102, 195404 (2020)

T7 Bálint Fülöp, Albin Márffy, Simon Zihlmann, Martin Gmitra, Endre Tóvári, Bálint Szentpéteri, Máté Kedves, Kenji Watanabe, Takashi Taniguchi, Jaroslav Fabian, Christian Schönenberger, Péter Makk, Szabolcs Csonka, Boosting proximity spin orbit coupling in graphene/WSe_2 heterostructures via hydrostatic pressure, npj 2D Mater Appl 5, 82 (2021)

T8 Tosson Elalaily, Martin Berke, Máté Kedves, Gergő Fülöp, Zoltán Scherübl, Thomas Kanne, Jesper Nygård, Péter Makk, and Szabolcs Csonka, Signatures of gate-driven out of equilibrium superconductivity in Ta/InAs nanowires, ACS Nano 17, 6, 5528–5535 (2023)

T9 Bálint Szentpéteri, Albin Márffy, Máté Kedves, Endre Tóvári, Bálint Fülöp, István Küke-mezey, András Magyarkuti, Kenji Watanabe, Takashi Taniguchi, Szabolcs Csonka, Péter Makk, Tuning the proximity induced spin-orbit coupling in bilayer graphene/WSe_2 he-terostructures with pressure, arXiv:2409.20062, submitted to Phys. Rev. B

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