Summary of „Dynamics of One-Dimensional Integrable Systems”

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The thesis focuses on the dynamics of one-dimensional integrable quantum systems, a special class of many-body models, distinguished by the existence of infinitely many conserved quantities. These conserved quantities prevent the systems from fully thermalizing in the conventional sense, instead allowing them to reach equilibrium states described by a Generalized Gibbs Ensemble (GGE), which accounts for the extra conservation laws, providing a comprehensive framework for predicting long-time behavior. In recent years, the field has also witnessed the development of Generalized Hydrodynamics (GHD), a powerful tool that leverages these conserved quantities to describe the transport properties and non-equilibrium steady states of integrable systems.

The thesis contributes to two central aspects of the theory of integrable quantum systems: deepening the theoretical foundations of GHD, through the study of current operators and exploring specific 'simple' integrable models that may offer potential for testing GHD's predictions against exact solutions.

The first part of the thesis rigorously establishes a formula for the mean values of current operators in the XXZ model, providing a foundational result for one of the key assumptions underlying GHD. Using a form factor expansion based on the algebraic Bethe Ansatz (ABA) and the solution of the quantum inverse scattering problem, current mean values are calculated for arbitrary eigenstates of the XXZ model in finite volume. These results are then extended to the XYZ model, where a generalized ABA approach and the algebraic construction of current operators yield a similar formula for the current mean values. This extension to non-$U(1)$ symmetric systems provides a first step towards establishing GHD‘s applicability across a broader class of integrable models.

The thesis also examines two specific 'simple' integrable models that share the common property of having particularly simple two-particle scattering phases. First, the folded XXZ model is studied extensively. The charges of this model are derived from those of the XXZ spin chain, its solution is constructed through coordinate Bethe Ansatz (CBA), its ground state properties are explored, and even the time-evolution of expectation values in specific quench scenarios are calculated, which is a task that is normally impossible in usual integrable systems. Notably, the folded XXZ model demonstrates Hilbert space fragmentation, a phenomenon where the system's Hilbert space divides into dynamically disconnected sectors leading to persistent oscillations. The thesis includes numerical simulations of the real-time dynamics following a global quantum quench, confirming the presence of persistent oscillations and non-thermal behavior. The same phenomenon is also observed in an integrability breaking extension of the folded XXZ model, with the level spacing statistics calculated to demonstrate departure from integrable behavior.

Finally, the thesis introduces and investigates a new anyon-like spin ladder model, representing one of the simplest interacting integrable systems. Through its relation to a specific Trotterization limit of the XXZ model, this model's integrability is established, its solution by means of CBA constructed and its entanglement properties are analyzed both by analytical and numerical tools, in equilibrium and out-of-equilibrium situations.

Together, these findings advance the theoretical understanding of non-equilibrium dynamics in integrable quantum systems, providing rigorous support for one of the underlying assumptions of GHD and potential new avenues for testing its predictions in the future.