Out-of-equilibrium dynamics in strongly correlated one-dimensional quantum many-body systems

PhD Thesis Booklet

Dávid Szász-Schagrin

Supervisor: Dr. Gábor Takács
Professor
BME Institute of Physics
Department of Theoretical Physics

MŰEGYETEM 1782

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Introduction

Understanding the out-of-equilibrium dynamics of interacting low-dimensional quantum many-body systems is a central objective of contemporary condensed matter physics, mainly due to the recent rise in the development of cold-atomic experiments. On the one hand, isolated quantum systems can thermalise very slowly, and understanding their dynamics requires experimental and theoretical study of their non-equilibrium setting. On the other hand, many of these systems are integrable, showcasing peculiar scattering properties that often allow for exact analytic results. Describing the non-equilibrium dynamics is particularly tasking: due to the low dimensionality of these systems, the role of quantum correlations is greatly enhanced. As a result, the interest in low-dimensional quantum systems such as (1+1)-dimensional quantum field theories and spin chains has considerably grown in recent years. One of the main questions is the equilibration of these systems and the full description of the relaxation and the steady state, as well as the role of integrability and its breakdown on the non-equilibrium dynamics.

This thesis focuses on the dynamics of one-dimensional interacting quantum systems. In particular, the weak breakdown of integrability, a novel class of integrability breaking where the breakdown occurs at higher-than-first order in perturbation theory. Moreover, I focus on the non-equilibrium time evolution of (1+1)d relativistic quantum field theories and investigate the validity of semi-classical approaches and non-equilibrium phenomena such as the decay of the false vacuum.

Aims

An important goal was to demonstrate that the weak integrability breaking of the XXZ spin chain can be captured in the level spacing statistics of the system by comparing weakly and strongly integrability breaking perturbations of the model. The notion of weak integrability breaking is novel, with very recent results and many open questions regarding the finite size scaling exponents characteristic of the different classes of integrability breaking.

Our general aim is the description of the out-of-equilibrium dynamics of interacting one-dimensional quantum field theories. Understanding the dynamics is far from trivial, and a widely used approach is the semi-classical approximation of the time evolution. Our concrete aim was to determine the range of validity of two different semi-classical approaches, the self-consistent Hatree-Fock approximation (SCA or mean field) and the truncated Wigner approach (TWA) by direct comparison of their time evolution to the more controlled truncated Hamiltonian approach (THA) in interacting quantum field theories.

A third goal was the real-time simulation of the false vacuum decay in the (1+1)d φ^4 theory and the accurate determination of the decay rate, a popular direction of condensed matter and high-energy physics in recent years due to its cosmological relevance. Our last objective was the precise understanding of the non-equilibrium time evolution of the sine-Gordon model, partially describing an experimental system of two coupled one-dimensional bosonic quasi-condensates. To correctly understand the physics of the experimental system, it is crucial to establish the simplest theoretical model that describes the dynamics, and requires the determination of relevant degrees of freedom in the experiments and understanding the limitations of the sine-Gordon description.

Methods

During my work, I applied analytical and numerical tools as well. To investigate the weak integrability breaking of the XXZ spin chain, I used numerical exact diagonalisation to compute the level spacing statistics of the system and study the finite size scaling of the crossover coupling. For the non-equilibrium dynamics of the $(1+1)$d φ^4 theory, I implemented the self-consistent Hartree-Fock approximation (SCA or mean field), a widely used semi-classical method to study the dynamics of interacting many-body systems. Moreover, I used the truncated Hamiltonian approach, a well-controlled method to study the dynamics of interacting field theories, serving as a benchmark for the semi-classical SCA and the truncated Wigner approach (TWA). The THA was also used to simulate the decay of the false vacuum in the theory that required a mini-superspace extension of the model to achieve the necessary accuracy. Additionally, to study the non-equilibrium dynamics in the sine-Gordon model, I developed a mini-superspace based truncated conformal space approach (MSTHA) and used it to validate the (TWA) as well.

New scientific results

I. **I demonstrated that the weak integrability breaking of the spin-1/2 XXZ spin chain induced by perturbing it with one of its conserved generalised currents can be captured in the level spacing statistics of the perturbed system.**

   It has been previously shown that perturbing an integrable spin chain by one of its higher conserved currents preserves the integrable properties of the system at first order in perturbation theory – in contrast to the usual 'strong' integrability breaking – therefore only 'weakly' breaking integrability.

   Applying exact diagonalisation and computing the level spacing statistics as an indicator of the integrability breaking, I compared the finite size scaling of the crossover coupling of the strongly integrability-breaking next-to-nearest-neighbour interaction and the current perturbation and demonstrated that the latter breaks integrability in the weak sense. My findings were published in [1].

II. **I determined the range of validity of two popular semi-classical approaches, the self-consistent Hartree-Fock approximation (SCA) and the truncated Wigner approach (TWA) by simulating quantum quenches in the (1+1)d φ^4 theory and comparing the non-equilibrium time evolution to the truncated Hamiltonian approach (THA).**

I demonstrated the applicability of the THA for studying the non-equilibrium dynamics by quenching the mass and interaction parameters and analyzing the cutoff dependence of the time evolution of the one-point function for a wide range of quench strengths. I demonstrated the failure of the semi-classical approximations for increasing interaction strength and the rise of a symmetry-broken steady state in the TWA anytime the bare mass becomes negative. These results were published in [2].

III. **I simulated the decay of the false vacuum and numerically determined the bubble nucleation rate in the spontaneously symmetry-broken (1+1)d φ^4 theory.**

Using the truncated Hamiltonian approach, I simulated the real-time decay following quantum quenches in a system initiated close to the false vacuum and extracted the decay rate. The numerical results agree with the theoretical predictions up to an overall (numerical) normalisation factor that only depends on the interaction coupling of the theory. Moreover, these results establish the THA as a powerful tool to investigate strongly non-perturbative phenomena, even in a non-equilibrium setting. I published my results in [3].

IV. **I simulated the non-equilibrium dynamics of the sine-Gordon model, partially describing a pair of Josephson-coupled one-dimensional bosonic quasi-condensates. The results establish that in the experimentally available weakly interacting regime the dynamics can be well approximated by a semi-classical description, and also clarify the role of the phononic modes of the theory.**

Using a novel version of THA based on a mini-superspace (MSTHA) and the truncated Wigner approximation, we simulated the time evolution of the one-point function and occupation numbers for a wide range of interaction strengths and two classes of quench protocols corresponding to small and large energy densities. Additionally, our results demonstrate that the MSTHA accurately describes the time evolution from the hard-core boson limit to the weakly interacting regime available in experiments when the injected energy density is sufficiently small. These findings were published in [4].

References

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