Theses of the Ph.D dissertation

Models of nuclear alpha cluster interacting with super-intense laser fields

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1 Introduction

The theoretical study of alpha decay is currently experiencing a renaissance. It is partly due to the increasing computational capacity enabling to investigate more accurately nuclear structural properties and nuclear stability in order to facilitate a more nuanced comprehension of inter-nucleon interactions. On the other hand, however, alpha decay has also come into the spotlight of research due to the recent advancement in the development of super-intense laser facilities that paves the way for new advancements in the investigation of laser-matter interactions. The experimentally accessible peak intensities (up to even $I = 10^{24}$ W/cm^2) with relatively low photon energies (less then 1 keV) could be incapacitated to alter even nuclear reaction rates or radioactive decay properties, such as the decay width of alpha decay. Among these, the potential modification of the decay width of alpha decay due to the influence of some intense laser field is a particularly intriguing question, as this process could be beneficial in the long term for the storage of radioactive waste.

Today, the literature covering the theory of laser-assisted alpha decay is gradually extending , nevertheless, due to the complexity of the fundamental process of alpha decay (understanding inter-nucleon interactions, nuclear pair correlations, clustering mechanisms and tunneling) and the diversity of the decay rates among the isotopes, as well as the lack of experimental results thus far, the models of laser-assisted alpha decay are highly approximate in nature (being based on quasi-classical approximations) and quite limited in terms of their accuracy and validity to isotopes. Therefore, there is a great need to develop a fundamental model framework that captures the essence of the problem and is quantum-mechanically consistent.

Alpha decay is a complex spontaneous nuclear process formed by the combined effects of nuclear forces acting between nucleons and electromagnetic interactions. It can be considered as a two-step process comprising the formation of the alpha cluster (clustering) and the interaction between the preformed alpha cluster and the residual nucleus; the latter resulting in the tunneling through the Coulomb barrier. It can be shown that, to leading order, the most pronounced effect of the laser field is expected in the modification of the Coulomb barrier, primarily influencing the tunneling process; the impact on the nuclear structure is not significant due to the strength of the nuclear forces.

In this dissertation, therefore, I investigate the tunneling phase of alpha decay in the case of heavy alpha-decaying isotone nuclei. After demonstrative calculations within the Wentzel-Kramers-Brillouin (WKB) approximation, I determine the decay width of the preformed alpha cluster represented by a quasi-stationary state, not in the conventional tunneling picture but in an alternative, quantum mechanically consistent non-hermitian model framework. This decay width is given by the imaginary part of the complex energy assigned to the quasi-stationary state, and I explore its modification due to an external laser field in leading order perturbation theory. Special attention is paid to the electrodynamic questions arising in the description of the interaction between the super-intense laser and the alpha-decaying nucleus, which become pronounced in the non-hermitian quantum mechanical model and within the non-relativistic approximation.

2 Main goal

The general aim was to establish an adequate and quantum-mechanically consistent theoretical base model for the description of super-intense laser-assisted alpha decay.

To achieve this, it was first necessary to conduct a demonstrative calculation, wherein the tunneling process is interpreted within the conventional quantum mechanical framework.

The goal was to show that, even under significantly simplified conditions—considering a plane-wave laser and calculating the laser-modified decay width in a zeroth-order approximation—the effect of the laser is expected to be non-negligible, indicating that further modeling efforts are justified.

Since the model based on the Henneberger transformation is consistently applicable only in the case of laser fields with periodic time-dependence, an alternative modeling framework was required to study the effects of laser pulses. The (t,t') formalism, which has been successfully applied in numerous areas of physics, provides an appropriate framework for the analytical and perturbative investigation of time-dependent, non-periodic potentials. However, the method relies on the knowledge of the wave-function characterizing the system, which, in the problem under examination, appears as a quasi-stationary state. A key feature of this state is the divergence of its wave-function in the spatial coordinate, which needs to be addressed in the calculations. This issue cannot be resolved within the conventional framework of Hermitian quantum mechanics, necessitating an extension of the description into the domain of non-hermitian quantum mechanics, which serves as the natural environment for the consistent treatment of quasi-stationary states (decaying systems). Accordingly, a consistent description of tunneling in alpha decay requires the formalism of non-hermitian quantum mechanics, along with its specialized tools such as *Complex Scaling*. In relation to this, an essential goal was the analysis of the complex spectrum of the complex-scaled Hamilton operator describing the system and the examination of its dependence on the parameters of the mean-field nuclear potentials.

The determination of the decay width of the quasi-stationary alpha cluster state through complex spectral calculations does not yield the total decay width, only that which is associated with the tunneling process. Thus, an important objective was the validation of the calculations, using the experimentally known decay lifetimes of specific isotone series.

The primary objective was to compute the first-order, non-relativistic (t,t')-perturbative complex energy correction for heavy alpha-decaying nuclei in the presence of circularly and linearly polarized laser pulses, taking into account and analyzing the intensity and photon energy dependence of the non-relativistic limit. An associated key aim of the dissertation was to highlight the limitations in the application of characteristic gauges (rE length-gauge and pA radiation-gauge)in the interaction of intense coherent electromagnetic fields with matter when providing a wavefunction-centered, non-Hermitian description of quasi-bound states.

3 Methods

In the first phase of this work, I investigated tunneling in an intense laser field using the conventional WKB approximation in a non-relativistic framework. During this, I rendered the time-dependence of the external laser field implicit through the *Henneberger transformation*, which allows for the determination of the zeroth-order, laser-modified decay width in the case of periodic vector potentials.

In the subsequent phase, I employed the methods of *non-Hermitian quantum mechanics* to determine the decay width of the alpha cluster described by a quasi-stationary state. As the principal method, I applied the *Complex Scaling transformation* to regularize the Gamow-Siegert functions characteristic of the quasi-stationary state. To obtain the complete complex spectrum of the non-Hermitian Hamiltonian operator describing the alpha cluster and the residual nucleus system and to determine the complex cluster energies, I utilized a symbolic diagonalization method.

The effect of an external, time-dependent, non-periodic vector potential characterizing the laser field was determined through a perturbative correction to the complex energy. For this purpose, I employed the (t,t')-formalism, specifically applicable in the case of non-Hermitian, decaying systems, and derived an analytical expression for the *first-order (t,t')-perturbative correction* to the complex energy.

4 New scientific results

Thesis 1

I investigated the theoretical possibility of laser-assisted alpha decay in the presence of an intense laser field ($I \geq 10^{19}$ W/cm^2) assuming a plane-wave laser. In the non-relativistic and long-wavelength approximation, using mean-field nuclear models, I calculated the relative change ($\mathcal{R} = \frac{\Gamma^{\text{las}}}{\Gamma}$) in the decay width ($\Gamma$) valid in the Wentzel-Kramers-Brillouin (WKB) approximation for the alpha-decaying isotope ^{210}Po in the Henneberger picture. I examined the dependence of $\mathcal{R}$ on the frequency (ω) and intensity (I) of the external laser field for both circularly and linearly polarized laser fields. I demonstrated that, for circularly polarized lasers, depending on the free parameters of the Woods-Saxon nuclear potential, the estimated zero-order value of the ratio lies between $\mathcal{R} = 30$ and $\mathcal{R} = 123$, indicating that the effect of the intense laser field on the decay width is already significant at the zeroth order of the Fourier expansion.

Related publication: [1]

Thesis 2

I developed a general method suitable for determining the change in the complex spectrum of a quasi-bound system in the leading order of perturbation theory in the presence of a non-periodic time-dependent potential. By combining the (t,t')-formalism with a numerical discretization method for determining the complex energy spectrum, I derived an analytical, closed formula for the Floquet-type energy eigenvalue correction $\left(\epsilon^{(1)}\right)$ in the leading order of perturbation theory, which can be trivially projected onto the actual energy correction. As a result of the method, I determined the change in the imaginary energy, interpreted as the change in the lifetime of the state, caused by different time-dependent Gaussian-type perturbing potentials.

Related publication: [2]

Thesis 3

I investigated the perturbative effect of an intense laser pulse, represented by a minimally coupled classical vector potential characterized by a Gaussian envelope function, on a quasi-bound system described by a special Gaussian-type potential barrier, in terms of the complex energy eigenvalue of the decaying system. Using the (t,t')-perturbation theory, I derived a general analytical formula for the first-order complex-energy correction, applicable to decaying systems described by continuous analytical potentials, in the field of either linearly or circularly polarized laser pulses. I established that, in the calculation of the first-order (t,t')-perturbative complex-energy correction, the non-zero matrix element contribution arises from the quadratic term in the vector potential $\left(\mathbf{A}^2\right)$ instead of the term proportional to $\mathbf{pA}$ that regularly appears in electromagnetic interactions.

Related publication: [3]

Thesis 4

I demonstrated that the non-Hermitian description of quasi-bound states interacting with an intense laser field, characterized by a classical vector potential, and specifically the application of the c-product, excludes the validity of the special (**pA**) and (**rE**) gauges for this problem. Furthermore, the "velocity-gauge" can be considered a physical gauge, consistent with the general description of systems interacting with intense laser fields under specific approximations (LWA and non-relativistic). I showed that the mathematical reason behind this is the requirement to apply the c-product on the complex-scaled system, which leads to a violation of the phase symmetry of the wave function. Consequently, the phase transformations of the wave function, as required by the full gauge symmetry, are only valid to a limited extent.

Related publication: [3]

Thesis 5

I performed an approximation of the complex energy spectrum of the Hamiltonian describing the quasi-stationary alpha-cluster state and the residual nucleus system, considering different mean-field nuclear potentials—specifically, the Woods-Saxon potential coupled to a Coulomb barrier and a Woods-Saxon corrected harmonic oscillator potential. Through a detailed analysis of the harmonic oscillator basis functions and the nuclear potential parameters, I identified the alpha energy (E_α) of the ^{212}Po alpha-decaying isotope with a numerical precision of approximately 1%. Furthermore, I determined the decay width characterizing the quasi-stationary state and demonstrated its relation to the total decay width of alpha decay.

Related publication: [4]

Thesis 6

I employed the analytical, complex Floquet type, first-order perturbative, non-relativistic energy-correction formula derived to laser pulses for the alpha-decaying isotones ^{212}Po, ^{214}Rn, ^{216}Ra, and ^{218}Th. I explored the dependence of the relative change of the lifetime on the key control parameters of the laser pulse, including peak intensity, photon energy, pulse duration, and phase shift. It was found that the pulse length exerts a dominant influence on the shift in the imaginary energy, which decreases with an increasing number of cycles. Additionally, the combined values of peak intensity and photon energy jointly determine the magnitude of this change. I provided an estimate of the boundaries of the non-relativistic approximation's validity in the context of alpha-cluster interactions with super-intense laser pulses. The results indicate that, at a photon energy of 100 eV, a peak intensity of 10^{24} W/cm^2 is still permissible, and the computed relative change of the lifetime is largest for ^{212}Po among the four isotones.

Related publication: [4]

Thesis 7

In the non-Hermitian model framework I developed, I determined the complex energy eigenvalue, with particular emphasis on the imaginary part, Γ, for short lifetime nuclei in the isotone series with $N = 128$, 130, and 132. I demonstrated that for the $N = 128$ isotone series, the ratio of the calculated Γ values to the experimentally known total decay widths (Γ^{decay}) is constant within 4% accuracy. This consistency confirms that for these isotonic nuclei, the dominant contribution to the difference in the decay widths, in the alpha-cluster and residual nucleus system is due to changes in the Coulomb barrier induced by the protons. Based on this, I provided an estimate for the half-life of the yet unobserved uranium isotope with $Z = 92$, $A = 220$, yielding $T = 141.7 \pm 5.3$ ns. I concluded that for alpha-decaying isotones, the empirical nature of the Geiger-Nuttall law can be adequately described by a single, non-perturbatively calculated quantity, the complex energy of the quasi-bound state describing the alpha cluster.

Related publication: [5]

5 Publications

[1] D. Kis and R. Szilvasi, *J. Phys. G: Nucl. Part. Phys.* **45, 045103** (2018)

[2] Réka Szilvási and Dániel P Kis, *J. Phys. A: Math. Theor.* **55, 275301** (2022)

[3] R. Szilvasi, D. P. Kis, *Results in Physics*, **Volume 54 107080** (2023)

[4] Réka Szilvási and Dániel P Kis: *J. Phys. G: Nucl. Part. Phys* **51, 055101** (2024)

[5] Réka Szilvási, István Andorfi and Dániel P Kis *Nucl. Phys. A* **1053 122968** (2025) published online in october 2024